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THE MEASUREMENT OF THE LIGHT OF STARS WITH A SELENIUM PHOTOMETER, WITH AN APPLICATION TO THE VARIATIONS OF *ALGOL*

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Some three years ago I became interested in the possible application of selenium to astronomical photometry, and with Dr. F. C. Brown, I began to experiment on selenium cells, with the hope of using them for accurate measures of the light of stars. The idea is an attractive one: the proposed method being to expose a selenium surface to the light of a star, focused by a large lens, and to note the change of resistance by means of a galvanometer: the brighter the star, the larger the galvanometer-deflection. In theory this is very simple, but at the outset we met some of the difficulties which confront everyone who tries to work with selenium. Other agencies than light affect the resistance, and apparently no experimenter has solved, to his own satisfaction, the mysteries of this peculiar element.

After some futile attempts to obtain results from starlight at the focus of a 12-inch refractor, we found that a selenium cell gave large effects when exposed directly to the moon. This led to a study of the variation of the moon's light throughout a lunation,[1] and it was also found that various selenium cells have widely different curves of color-sensibility.[2] One reason that physicists and electricians have not used these cells in photometry is that selenium is relatively most sensitive to red light, but in certain kinds of

[1] *Astrophysical Journal*, 26, 326, 1907. [2] *Ibid.*, 27, 183, 1908.

astronomical work, such as the study of eclipsing variable stars, it is quite immaterial what part of the spectrum is used.

During several years of experimenting I have tried the selenium cells of different makers, whose processes are secret, also a number of cells made by Dr. Brown and myself. The best cells known to me are those by Giltay of Delft, Holland. In the form which he uses, two wires are wound in a double spiral about a flat insulator, and the spaces on one face are filled with selenium which is properly treated to make it sensitive. In response to my inquiry, Mr. Giltay has said that his method of sensitizing is indeed a secret, but he adds that it is often a secret even to himself, for after thirty years of experience in making cells he frequently has surprises, and usually of the disagreeable kind.

In working with faint lights, we found that a small variation in the temperature of a cell gives a greater change of resistance than is produced by the light of the brightest stars. I made numerous attempts to maintain a constant temperature with thermostats, but although some improvement was noted, the irregularities remained almost hopelessly large. One of the fundamental obstacles is the heating effect of the current which is necessarily passed through the selenium to measure the resistance. It is unnecessary to enumerate the various experiments which were made, but after a long time I learned that there are three simple precautions necessary to insure success, and they are given in what seems to be the relative order of importance:

1. The selenium should be kept at a uniform, low temperature, 0° C., or lower.

2. The current should pass continually through the selenium.

3. Exposures to light should be short, say 10 seconds, with longer intervals for recovery.

Such success as I have secured has been through the observance of these simple precepts, and although other experimenters have not formulated such rules, I have been quite unable to do anything in measuring starlight without adhering to them. In discussing the reasons for these precautions, it is not possible to give numerical results which are applicable to different cells, but the general principles seem to hold wherever faint lights are to be measured.

First, the effect of low temperature is best illustrated by an example. A cell of about 1,000,000 ohms resistance at 20° C. increases to 3,000,000 ohms at 0° C. It is found that small variations in temperature produce relatively much less effect upon the higher resistance, and, in terms of galvanometer-deflections, the irregularities are in the ratio of about 50 to 1 in favor of the low temperature. Also, the light-action is about twice as great at 0° as at 20°, and we have the rather astonishing result that it is possible to increase the effective sensibility of a cell 100 fold merely by reducing its temperature. In practice it is not difficult to surround a cell with an ice-pack, but moisture must be kept from the cell and from the insulation of the connecting wires.

Second, the current which passes through the selenium should not be interrupted. If an E.M.F. of 6 volts be applied to a cell of 3,000,000 ohms, the resistance decreases slowly, until at the end of half an hour a steady condition is reached where the resistance is about 10 per cent less than at the start. This change is presumably due to the heating of the current, which is enough to account for the decrease. In terms of galvanometer-deflections, this heating effect is of the order of 100 times the light-effect from a bright star, and therefore the cell should not be disturbed by breaking the circuit.

Third, as is well known, selenium requires some time for recovery from light-action, and it is found that if a cell be exposed for 10 seconds to starlight, it is best to wait about 1 minute to insure recovery and allow the cell to regain its original sensibility.

Other precautions are the same as must be taken in any work where high resistances and a sensitive galvanometer are used. Good insulation is essential, and the connecting wires must be shielded from the effect of static charges, which are especially large in cold weather.

As the character of the galvanometer used is of considerable importance, I shall give the constants of the one I have at present, which was made by the Weston Electrical Instrument Co. It is of the d'Arsonval type, with flat mirror 8 mm in diameter, and is used with a view telescope magnifying 25 times, and a millimeter scale placed at from 2 to 4 meters distance. The resistance is

512 ohms. The period of complete free swing is 5.0 seconds, and when connected in circuit the instrument is practically dead beat. The "figure of merit," or current required to produce 1 mm deflection at 1 meter scale-distance, is 2.4×10^{-10} amperes, but because of the excellence of the optical system a current of 5×10^{-12} amperes can be detected with certainty. Although this sensibility is not the highest possible in a galvanometer, it is the best I have ever seen in one of the d'Arsonval type, and there is the great advantage that the instrument is practically unaffected by external conditions.

In the arrangement for measuring starlight the selenium cell is connected as one arm of a Wheatstone bridge. Current is supplied by a few dry cells, which are ample for the purpose. The resistances of the selenium cell, galvanometer, and battery being fixed, the other arms of the bridge should be chosen to give the maximum current through the ga vanometer for a small change in the resistance of the selenium. Convenient formulae for the best arrangement have been derived by Heaviside;[1] but as megohm resistance boxes are somewhat expensive, I have found that with resistances of 10,000 ohms and less it is possible to obtain fully three-fourths of the computed maximum sensibility. Precaution must be taken that the resistances are so arranged that the galvanometer is not damped excessively, in order that full deflection may be secured.

Fig. 1 shows the ice chamber containing a cell, attached to our 12-inch telescope. In warm weather the ice is renewed every day, but in winter the end of the telescope is wrapped up in a blanket, and may not be disturbed for a month at a time. Exposures to starlight are made by opening a small shutter which is operated by hand, the observer listening to the beats of a chronometer. In Fig. 2 are shown the galvanometer, scale, and resistance boxes which are located in a room some distance from the dome, the two wires leading from the telescope being easily carried to where it is convenient to mount the galvanometer. In practice we always have two observers, one to point the telescope and make the exposures, while the other reads the galvanometer and records

[1] *Philosophical Magazine*, **45**, 114, 1873.

the measures. Contrary to astronomical custom, the recorder is the real observer in this work, and the assistant is stationed at the telescope. In fact, it often happens that the man at the galvanometer can tell that the sky is growing less transparent, before the observer in the dome can ascertain the same by direct inspection.

This brief outline of the method of using selenium in photometry will perhaps indicate the general way in which the observations are made, but the procedure will be better illustrated by giving the actual figures and results derived in an exhaustive study of a variable star.

APPLICATION TO THE LIGHT-VARIATIONS OF *ALGOL*

As soon as I began experimenting with selenium, I had it in mind as one of the problems to take up, when it became possible to measure starlight with accuracy, to investigate the light-curve of *Algol*, especially for a secondary minimum, which, if present, would give us valuable information concerning the companion. The history of the investigations of *Algol* is so well known, even to the general reader, that it is superfluous to mention the names of the astronomers who have contributed to our knowledge of this star. In fact, we may say that if any star in the whole sky has been subjected to an exhaustive study, *Algol* is that one. For this very reason, however, it seemed advisable to try the selenium photometer, for this star would furnish an excellent test of the improvement of our work over visual methods.

It was found on trial that with good conditions a 10-second exposure of our best selenium cell to *Algol* at maximum light gave a scale deflection of 8.0 mm. The probable error of a single deflection was of the order of 0.16 mm, or about 2 per cent, and as this was a better accordance than can be secured in visual work, I felt ready to adopt a program for observing this star during several months. In illustration of the accuracy which was insisted upon, it may be said that when the cell happened to become so irregular that the probable error of one deflection became higher than 5 per cent, observations were not undertaken.

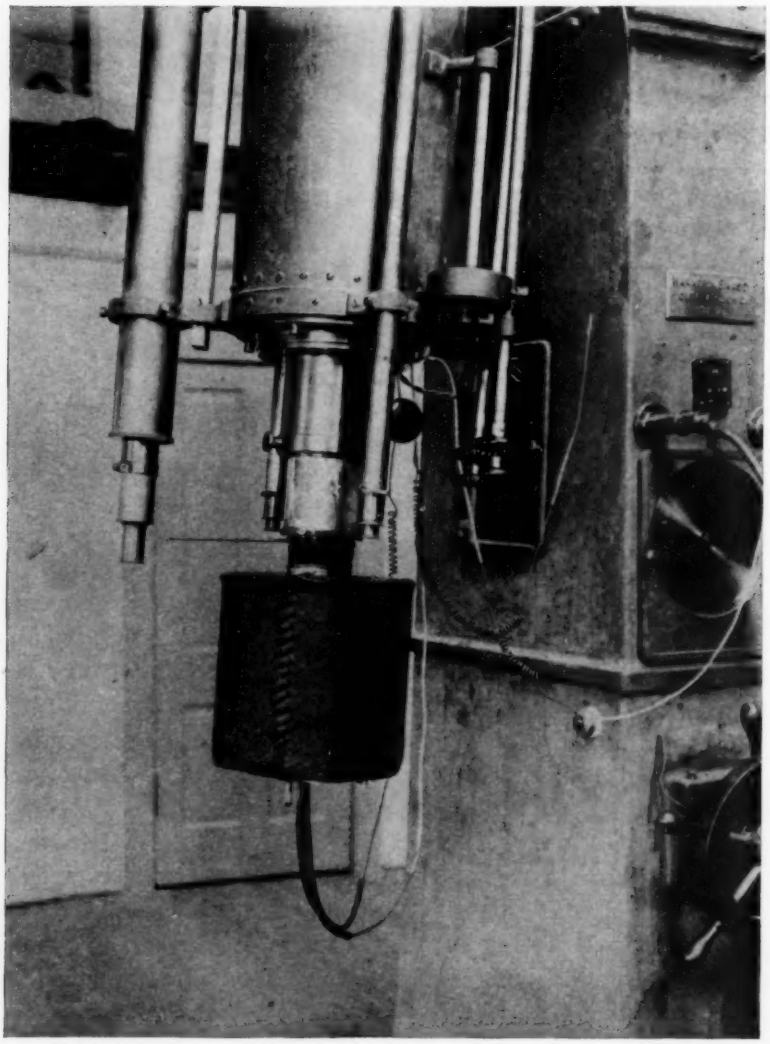

FIG. 1.—Selenium Cell in Ice-Pack Attached to 12-Inch Refractor

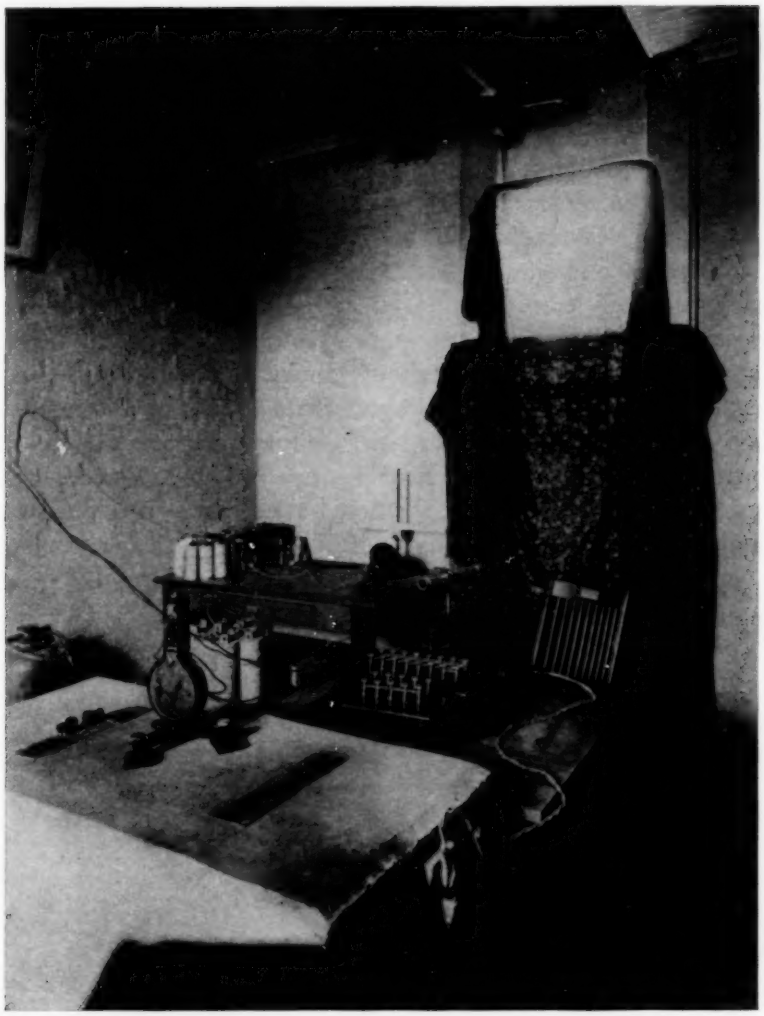

FIG. 2.—Galvanometer and Accessories for the Selenium Photometer

On account of the atmospheric absorption, there is of course no method of measuring the light of a star, which does not include observations of one or more comparison stars. On consideration, I decided to use *α Persei* and *δ Persei*. After making this decision, I found that these same stars had been chosen by Müller,[1] whose determination of the light-curve is perhaps the best that has ever been made visually. The normal plan with selenium was to make 4 exposures on *α Persei*, then 8 on *Algol*, and finally 4 on *α Persei*. Such a series of 8 readings on each star will be called a set. When *Algol* was near minimum, both *α* and *δ* were used, and a set of readings usually consisted of 4 on *α*, 4 on *δ*, 8 on *Algol*, 4 on *δ*, and 4 on *α*. When only two stars were observed, the 16 deflections of a set were secured in about 20 minutes, including the time for moving the dome and telescope. Ordinarily it was considered that 4 to 6 sets of measures were sufficient for one night, but when the phase of *Algol* was near principal minimum, or the supposed secondary minimum, the observations were continued all night, or as long as conditions would permit. The method of observation and reduction is best illustrated by an actual sample, and the following represents a typical page of the notebook.

FRIDAY, JANUARY 7, 1910

Observers, J. S. and P. F. W.
Cell, Giltay No. 93, area 18 × 26 mm.
Extra-focal star image, 7 mm diameter.
Ratio arms, 10,000 and 10 ohms.
Resistance of cell, 5,000,000 ohms.
Dome temperature, −5° C.
Galvanometer, Weston.
Scale-distance, 2.3 meters.
Battery, 7 Edison cells, 6 volts.
Exposures, 10 seconds.
Sky, fair; absorption factor, 2.
Wind, south.
Watch ΔT, +4 seconds.

[1] *Astronomische Nachrichten*, **156**, 178, 1901.

Time	Readings	Deflections
	α Persei	mm
$8^h\ 18^m$	280.0–87.3	7.3
19	281.7–88.9	7.2
20	282.9–90.2	7.3
21	284.4–91.6	7.2
		7.25
	Algol	
$8^h\ 24^m$	278.0–84.7	6.7
25	279.9–86.4	6.5
26	281.9–88.2	6.3
27	283.3–89.7	6.4
28	284.3–90.4	6.1
29	285.2–91.3	6.1
30	285.7–92.1	6.4
31	286.8–93.1	6.3
		6.35
	α Persei	
$8^h\ 34^m$	282.1–90.0	7.9
35	285.6–93.3	7.7
36	288.3–96.0	7.7
38	290.2–97.8	7.6
		7.72
		7.25
		Mean 7.48

REDUCTION

Mean time	$8^h\ 28^m$	
Sidereal time	3 42	
Star	*α Persei*	*Algol*
Deflection, D	7.48	6.35
Drift	−0.02	−0.02
Corrected D	7.46	6.33
Log. D	0.873	0.801
Log. ratio	0.072	
Difference of magnitude	0.18	
Absorption	0.00	
Corrected difference	0.18	

RESULTS ON JANUARY 7, 1910

Time	Difference of Magnitude	Residual	Time	Difference of Magnitude	Residual
$8^h\ 28^m$	0.18	0.00	$9^h\ 26^m$	0.16	−0.02
8 47	0.17	−0.01	10 00	0.22	+0.04
9 07	0.20	+0.02	10 19	0.16	−0.02
Means 8 47	0.18		9 55	0.18	±0.02

The ratio of the scale-deflections gives at once the difference of magnitude of the two stars, the main corrections which need to be applied being due to progressive drift of the galvanometer, and differential atmospheric absorption. At the beginning of each night's work, a series of readings was taken to determine the rate of change of the galvanometer zero, due to small temperature-variation in the selenium cell, this being repeated about once an hour, and the drift was then interpolated for the times of the measures of the stars. Near minimum, the difference in the deflections between *Algol* and *α Persei* was so large that the drift was of much consequence, and measures of *δ Persei* were utilized for the determination of drift. From the observations on a number of nights, the difference of magnitude between *α* and *δ Persei* was found to be 1.10 magnitudes, and this value was used for computing the drift.

The correction for atmospheric absorption was based upon the Potsdam tables by Müller.[1] A table of the correction was computed for *Algol*, *α* and *δ Persei* for each half hour of sidereal time, and the interpolated value from this table was multiplied by the factor, 1.0, 1.5, or 2.0, depending upon my judgment of the transparency of the sky. This may seem like a crude method, but in four-fifths of the cases the correction was either 0.00 or 0.01 magnitude. When *Algol* became relatively low in the west, additional observations for the absorption-factor were taken on *β Andromedae*, but there were only 3 sets of measures where the correction was greater than 0.04 magnitude.

Near the time of full moon, the bright background of the sky gives an appreciable deflection of the galvanometer; but where necessary this effect was carefully determined, and allowed for in the reductions.

Table I contains the results of observations which were taken of *Algol* from September 1909, to March 1910. The heliocentric phase was derived from Hartwig's ephemeris, which is based upon Chandler's elements. After combining the observations near principal minimum, I found a correction to the ephemeris of $-1^h\,16^m$, and this has already been applied in Table I. The

[1] *Die Photometrie der Gestirne*, Leipzig, 1897, p. 515

TABLE I

OBSERVATIONS OF *ALGOL* NEAR PRINCIPAL MINIMUM

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude
September 9,'09	18^h22^m	$+1^h11^m$	1.06	September 29,'09	22^h16^m	$+3^h24^m$	0.36
	19 04	+1 53	0.66	October 25	15 37	+1 27	1.13
	19 22	+2 11	0.68		16 14	+2 04	0.73
	19 56	+2 45	0.40		16 51	+2 41	0.59
	20 22	+3 11	0.43		17 20	+3 10	0.40
	21 08	+3 57	0.23		17 59	+3 49	0.30
	21 26	+4 15	0.21		18 26	+4 16	0.22
	21 56	+4 45	0.16	October 28	16 25	+5 26	0.17
September 12	16 42	+2 42	0.66		16 44	+5 45	0.18
	17 13	+3 13	0.46		17 06	+6 07	0.21
	17 44	+3 44	0.37		17 25	+6 26	0.18
	18 10	+4 10	0.25	November 14	13 36	−2 16	0.63
	18 28	+4 28	0.25		14 05	−1 47	0.87
	18 53	+4 53	0.16		14 35	−1 17	1.03
September 15	16 36	+5 48	0.14		15 09	−0 43	1.26
	17 14	+6 26	0.12		15 41	−0 11	1.39
	17 34	+6 46	0.12		16 14	+0 22	1.34
	17 53	+7 05	0.21		16 44	+0 52	1.19
September 26	16 40	−5 23	0.14		17 15	+1 23	1.11
	17 11	−4 52	0.24	December 7	13 07	−1 18	1.01
	17 48	−4 15	0.16		13 45	−0 40	1.20
	18 09	−3 54	0.31		14 18	−0 07	1.35
	18 48	−3 15	0.40		14 51	+0 26	1.36
	19 32	−2 31	0.61		15 29	+1 04	1.06
	20 12	−1 51	0.76		16 15	+1 50	0.84
	21 24	−0 39	1.17	February 5,'10	13 48	−5 52	0.17
September 29	17 35	−1 17	0.95		14 06	−5 34	0.14
	18 03	−0 49	1.28		14 25	−5 15	0.20
	18 46	−0 06	1.30		14 43	−4 57	0.19
	19 19	+0 27	1.42		15 05	−4 35	0.22
	19 50	+0 58	1.01		15 23	−4 17	0.22
	20 18	+1 26	0.95		15 39	−4 01	0.23
	21 11	+2 19	0.66		15 53	−3 47	0.30
	21 41	+2 49	0.45				

OBSERVATIONS NEAR SECONDARY MINIMUM

September 16,'09	18 45	31 57	0.13	October 15,'09	18 37	39 41	0.12
	19 07	32 19	0.15		18 55	39 59	0.10
	19 28	32 40	0.14		19 13	40 17	0.11
	19 56	33 08	0.17	November 4	15 21	34 44	0.19
	20 24	33 36	0.18		15 43	35 06	0.15
	20 42	33 54	0.16		16 05	35 28	0.19
	21 00	34 12	0.20		16 39	36 02	0.18
	22 00	35 12	0.20		16 59	36 22	0.19
	22 20	35 32	0.15		17 19	36 42	0.16
September 25	16 08	38 54	0.15		17 47	37 10	0.12
	16 28	39 14	0.14		18 15	37 38	0.14
	16 50	39 36	0.12	November 24	19 58	37 38	0.13
	17 09	39 55	0.17		20 16	37 56	0.14

TABLE I—*Continued*

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude
November 24,'09	20^h35^m	38^h15^m	0.16	December 20,'09	16^h42^m	39^h00^m	0.10
	20 53	38 33	0.14		17 00	39 18	0.12
	21 12	38 52	0.14	January 6, '10	15 07	32 29	0.15
December 17	13 12	32 19	0.16		15 39	33 01	0.15
	13 30	32 37	0.13		16 43	34 05	0.14
	13 51	32 58	0.21		17 07	34 29	0.19
	14 11	33 18	0.14		17 34	34 56	0.19
	14 45	33 52	0.24	February 1	12 28	34 27	0.16
	15 06	34 13	0.23		12 46	34 45	0.22
	15 28	34 35	0.21		13 05	35 04	0.20
	17 57	37 04	0.18		13 23	35 22	0.19
	18 16	37 23	0.14		13 42	35 41	0.17
	18 40	37 47	0.11		14 00	35 59	0.17
	19 04	38 11	0.16		14 31	36 30	0.15
	19 27	38 34	0.14		14 50	36 49	0.17
December 20	11 49	34 07	0.17		15 15	37 14	0.17
	12 11	34 29	0.18		15 34	37 33	0.13
	12 34	34 52	0.21		15 53	37 52	0.15
	12 52	35 10	0.19	February 18	14 32	31 34	0.12
	13 28	35 46	0.20		14 52	31 54	0.15
	13 47	36 05	0.17		15 31	32 33	0.15
	14 11	36 29	0.12		15 55	32 57	0.17
	14 32	36 50	0.18		16 11	33 13	0.19
	14 56	37 14	0.18	March 13	13 12	31 39	0.18
	15 36	37 54	0.13		13 25	31 52	0.14
	15 57	38 15	0.12		13 46	32 13	0.18
	16 15	38 33	0.13		14 15	32 42	0.20

OBSERVATIONS BETWEEN MINIMA

Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Number of Sets	Date	G.M.T.	Heliocentric Phase	Difference of Magnitude	Number of Sets
Sept. 10,'09	17^h15^m	24^h04^m	0.13	2	Nov. 29,'09	13^h54^m	13^h56^m	0.17	3
11	18 52	49 41	0.14	3	29	15 01	15 03	0.17	3
13	16 58	26 58	0.15	4	Dec. 8	14 02	23 37	0.14	3
17	17 29	54 41	0.16	4	8	15 13	24 48	0.13	3
23	17 02	60 36	0.18	4	9	13 55	47 30	0.12	3
24	16 45	15 31	0.14	4	9	15 00	48 35	0.14	3
27	17 34	19 31	0.15	4	18	15 33	58 40	0.17	3
28	16 57	42 54	0.15	4	18	16 40	59 47	0.17	3
Oct. 1	16 44	45 52	0.12	4	19	12 12	10 30	0.15	3
6	17 26	28 57	0.14	4	19	13 13	11 31	0.15	3
14	16 00	13 04	0.17	3	Jan. 7,'10	14 47	56 09	0.18	3
18	17 32	41 49	0.14	4	7	15 55	57 17	0.18	3
21	15 52	43 20	0.13	4	8	12 34	9 08	0.17	3
21	17 29	44 57	0.13	4	8	13 42	10 16	0.17	3
29	16 06	29 07	0.13	4	31	14 36	12 35	0.17	3
30	16 02	53 03	0.16	4	Feb. 12	13 43	24 24	0.13	3
Nov. 3	17 00	12 23	0.14	4	12	14 21	25 02	0.13	3
5	16 49	60 12	0.15	4	19	13 31	54 33	0.15	3

difference of magnitude is uniformly in the sense: magnitude of *Algol* minus magnitude of *α Persei*. Near minima the result for each set is given, but between minima, where there could be no rapid change in light, I have averaged the several sets of each night.

The observations of Table I were arranged according to phase, and each successive 3 or more sets were combined, the normal magnitudes containing sets as follows:

Near principal minimum ..... 3 sets
Near secondary minimum ..... 6 sets
Between minima ..... 9 or more sets

It was afterward found that the duration of the eclipsing phase is less than 10 hours, and the first and last normals near principal

TABLE II

NORMAL MAGNITUDES

NEAR PRINCIPAL MINIMUM			NEAR SECONDARY MINIMUM		
Phase	Difference of Magnitude	O.−C.	Phase	Difference of Magnitude	O.−C.
-5^h18^m	0.18	+0.01	31^h52^m	0.150	0.000
−4 22	0.20	−0.02	32 30	0.147	−0.014
−3 54	0.28	0.00	33 00	0.182	+0.012
−2 41	0.55	0.00	33 49	0.172	−0.011
−1 39	0.88	−0.01	34 24	0.195	+0.007
−1 08	1.09	0.00	34 54	0.193	+0.005
−0 41	1.21	−0.05	35 24	0.182	−0.002
−0 08	1.35	−0.02	36 07	0.172	−0.003
+0 25	1.37	+0.04	36 51	0.160	−0.003
+0 48	1.15	−0.06	37 27	0.148	−0.004
+1 13	1.08	+0.02	37 59	0.142	0.000
+1 34	0.97	+0.04	38 37	0.137	+0.004
+2 03	0.69	−0.06	39 38	0.122	−0.006
+2 34	0.64	+0.05			
+2 55	0.42	−0.07			
+3 16	0.42	+0.02	BETWEEN MINIMA		
+3 50	0.30	+0.01			
+4 14	0.23	0.00	9^h58^m	0.163	−0.002
+4 42	0.19	+0.01	12 10	0.153	−0.007
+6 14	0.17	0.00	14 01	0.170	+0.012
			19 33	0.143	−0.002
			24 25	0.130	−0.005
			27 01	0.138	+0.006
			42 41	0.140	+0.008
			46 06	0.123	−0.017
			50 26	0.147	−0.001
			55 40	0.168	+0.008
			59 49	0.168	0.000

minimum include 6 sets of measures. In Table II are listed the normal magnitudes, together with the residuals formed by a comparison with the final light-curve computed from the elements of the system of *Algol* which I have derived.

Except near principal minimum, the accordance of the observations justifies carrying the normal magnitudes to the third decimal, and it will be seen that the accuracy obtainable with the selenium photometer, for bright stars at least, is greater than has been attained in any visual or photographic method.

From the residuals in Table II we have

Probable error of a normal magnitude near principal minimum $= \pm 0.023$ mag.
Probable error of a normal magnitude near secondary or between minima $= \pm 0.006$ mag.

One peculiarity of the selenium photometer is that the errors, expressed in magnitudes, increase for faint stars, but in light-units the accordance is practically the same for any intensity. In visual observations, the residuals in magnitudes are about the same over a wide range of brightness. Where a variable star has a range of 1 magnitude and eye observations are expressed in light-units, the probable error near minimum is only $\frac{2}{5}$ that at maximum, and according to the usual method the weights should be assigned as 6 to 1. So far as I know, no computer has taken this fact into account, although there have been many elaborate calculations of the elements of different variable stars. In this connection, I may say that I am quite unable to see the justification for publishing the elements of a system like that of *Algol* to 4 or even 5 significant figures, when the original data are often not exact to 2 places, and when even the first figure of some of the results may be in error.

The normal magnitudes of Table II are shown graphically in Fig. 3. The first peculiarity of the light-curve which will attract those familiar with *Algol* is the existence of a secondary minimum. This has been sought for in vain by visual observers, though in my opinion if *Algol* were not so bright, the variation of 0.06 magnitude might have been detected. Of perhaps equal interest is the continuous variation in the light between minima, showing this star to be a distant relative of β *Lyrae*, though the maximum brilliancy

does not seem to occur midway between minima, the curve being highest just before and after secondary minimum. The fact that

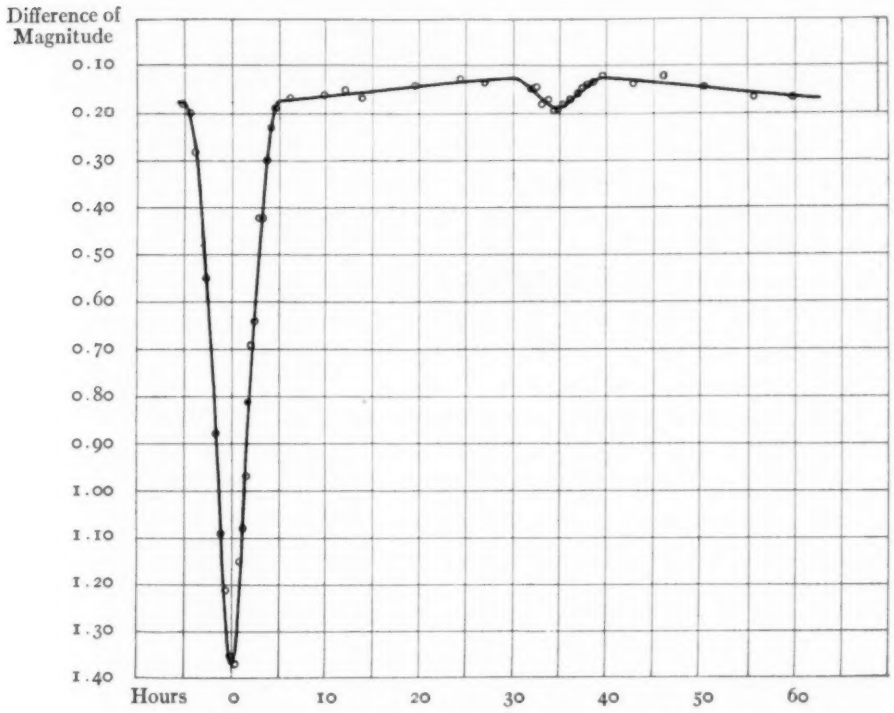

FIG. 3.—The Light-Curve of *Algol*

a smooth, symmetrical light-curve fits in with the observations is perhaps sufficient reason for believing this continuous variation to be real.

THEORY OF THE SYSTEM OF *ALGOL*

On the basis of the observations which have been tabulated, let us now consider what conditions in the system of *Algol* will account for the variations in light. Since Vogel's classic determination of the radial velocities before and after light-minimum, the eclipse theory has been regarded as established. The presence of the secondary minimum now proves that the companion is not wholly dark, and it is evident that we have to deal with a bright

and a relatively faint body, which in what follows I shall designate as *Algol* and the companion.

As a first approximation let us assume that the orbit is circular, that both bodies are spheres, and further that they are devoid of extensive atmospheres which would cause a decrease of intensity from center to limb, such as we know does exist in the case of the sun. The most obvious explanation of the continuous variation of light between minima is that the companion keeps one face toward *Algol*, and is brighter on that side due to radiation received from the primary. I therefore assume that the companion rotates uniformly once in the period of revolution, and that it is divided into two hemispheres each uniformly intense. While this last assumption is probably far from the truth, it is sufficient for the accuracy of the observations.

In the calculations which follow, I have converted stellar magnitudes into relative light, changing the unit as convenient. First, taking the light of *a Persei* as unity, let

ϕ = the angular phase, or true anomaly counted from minimum,
L = the corresponding total light of the system, as seen from the earth,
L_1 = the light due to *Algol* + the faint hemisphere of the companion,
$2s$ = the excess of light emitted by the bright hemisphere over the faint hemisphere of the companion.

It then easily follows that

$$L = L_1 + s(1 - \cos \phi). \qquad (1)$$

Each normal magnitude between minima in Table II furnishes a value of L for the corresponding ϕ, and solving the 11 equations by the method of least squares, I obtain for the two unknowns:

$$L_1 = 0.8507 \pm 0.0033,$$
$$s = 0.0201 \pm 0.0029,$$

or in terms of stellar magnitude, referred to *a Persei*, we have

$$L_1 = 0.176 \text{ mag.} \pm 0.004 \text{ mag.}$$
$$s = 0.025 \text{ mag.} \pm 0.003 \text{ mag.}$$

Using these values of L_1 and s, the light-curve between minima was computed, and is shown in Fig. 3. If we assume no variation between minima, the squares of the residuals give $[vv] = 0.001691$,

and the addition of the term in s reduces this to $[vv]=0.000470$, or to the order of one-fourth.

It is somewhat tempting to introduce another unknown which will express the ellipticity of *Algol* due to tidal action, but the residuals between minima are now so small that I should consider the result of such a computation as illusory. For the present it seems justifiable to state that, within the limits of error, the observations are represented by the simple assumption that both bodies are spheres, and that the companion is uniformly bright over each of the two hemispheres.

We are now ready to take up the elements of the system of *Algol*, as they may be determined from the variation throughout the eclipsing phase. In 1880 Professor E. C. Pickering[1] first worked out a determination of the relative distance and dimensions of the components of *Algol*, and his theory was extended by Harting.[2] I shall follow the method of Harting, with some changes necessitated by the influence of the light of the companion.

Let the radius, the surface-intensity, and the total light of *Algol* each be taken as unity, then the elements to be determined are:

κ = the radius of the companion,
r = the radius of the relative orbit,
i = the inclination of the orbit,
λ = the surface-intensity of the faint hemisphere of the companion,
$\lambda+\lambda_1$ = the surface-intensity of the bright hemisphere of the companion.

During the eclipse, we may consider the disks of the two stars projected on a plane perpendicular to the line of sight, and in this projection let

2η = the angle at the center of *Algol*, subtended by the common chord,
2ξ = the similar angle at the center of the companion,
ρ = the distance between centers,
M = the area common to the projected disks,
v = the projection of the true anomaly in the apparent orbit,
P = the period = 68.816 hours,
t = the time in hours from minimum,
L = the total light as seen from the earth.

[1] *Proceedings of the American Academy of Arts and Sciences*, **16**, 1, 1880.

[2] *Untersuchungen über den Lichtwechsel des Sternes β Persei*, Munich, 1889.

Then the following equations give the light L at any instant, when the elements are known:

$$\begin{aligned}
\phi &= 360^\circ \frac{t}{P} \\
\operatorname{ctn} v &= \cos i \operatorname{ctn} \phi \\
\rho &= r \frac{\sin \phi}{\sin v} \\
\cos \eta &= \frac{1+\rho^2-\kappa^2}{2\rho} \qquad (2) \\
\sin \xi &= \frac{1}{\kappa} \sin \eta \\
M &= \kappa^2 \xi - \rho \sin \eta + \eta \\
L &= 1 - \frac{M}{\pi} + \kappa^2 \lambda, \text{ near principal minimum,} \\
&= 1 - \frac{M}{\pi}(\lambda+\lambda_1) + \kappa^2(\lambda+\lambda_1), \\
&\qquad \text{near secondary minimum.}
\end{aligned}$$

In practice it is best to assume a set of elements, compute L for the phase of each normal magnitude, and from the residuals derive corrections to the elements. Confining ourselves to the principal eclipse, the corrections $\delta\kappa$, δr, and δi are given by equations of the form

$$\frac{\pi}{2}\delta L = \kappa(\pi\lambda - \xi)\delta\kappa + \frac{\rho \sin \eta}{r}\delta r - \frac{r^2 \sin 2i \cos^2 \phi \sin \eta}{2\rho}\delta i. \qquad (3)$$

From equations (2) and (3) the elements may be derived after λ is known. We can now make use of the observations outside of the principal eclipse, and let L with different subscripts represent the light received from the system by an observer on the earth; also put $M = M_0$ at either minimum. We have then

$L_1 = 1 + \kappa^2\lambda$, *Algol*+faint side of companion,

$L_2 = 1 + \kappa^2(\lambda+\lambda_1)$, *Algol*+bright side of companion,

$L_M = 1 + \kappa^2\lambda - \frac{M_0}{\pi}$, at principal minimum,

$L_m = 1 + \kappa^2(\lambda+\lambda_1) - \frac{M_0}{\pi}(\lambda+\lambda_1)$, at secondary minimum.

On referring to (1) it is seen that with the proper unit

$$\kappa^2\lambda_1 = 2s.$$

The values of L_1 and L_2 are already known in terms of the light of *a Persei*. L_M and L_m may be found graphically, but when approximate elements are known, $M_\circ$ and L_M are derived from the light-ephemeris.

The solution of the foregoing equations gives

$$\frac{M_\circ}{\pi} = L_1 - L_M$$

$$\lambda_1 = \frac{L_2 - L_1}{\kappa^2} \tag{4}$$

$$\lambda = \frac{\pi(L_2 - L_m)}{M_\circ} - \lambda_1 .$$

The unit of light to which the L's must be referred is defined by

$$L_1 = 1 + \kappa^2\lambda ,$$

and it was found from (1) that L_1 corresponds to a light which is 0.176 magnitude fainter than *a Persei*. Hence, each successive value of κ or λ requires that all computed and observed lights be referred to a new unit.

The approximation of the elements by (2), (3), and (4) is then as follows. Obtain preliminary values of κ, r, and i and substitute κ in (4), which gives λ_1 and λ; compute by (2) the light-ephemeris for a number of epochs, and form the residuals, δL; find the corrections $\delta\kappa$, δr, and δi from three or more observation equations (3); substitute the corrected κ in (4), changing the light-unit, and the new values for λ_1 and λ complete the set of elements. The ephemeris is now recomputed, and the process of correction repeated until satisfactory elements are secured.

Some valuable suggestions for finding preliminary values for κ, r, and i may be found in a paper by Schlesinger.[1] In the first approximations it is unnecessary to use all of the observations to form equation (3), perhaps the easiest method being to draw a light-curve and select 3 points which are sufficient to give κ, r, and i. When sufficiently approximate elements have been derived,

[1] *Publications of the Allegheny Observatory*, 1, 123, 1909.

the final solution may be made, using all of the observations, and forming normal equations in the usual way.

In applying this method to the case in hand I first assumed that the light-curve is symmetrical near principal minimum, and I arranged the normal magnitudes in Table II in order, without regard to the sign of the phase. Averaging each successive pair of normals, I obtained the results in Table III. The phase is now given in decimals of an hour, and the light is referred to difference of magnitude 0.244 as unity, which is the final adopted brightness of *Algol*. The residuals are derived from a comparison with the light-ephemeris computed from the final elements.

TABLE III

COMBINED NORMAL MAGNITUDES NEAR PRINCIPAL MINIMUM

Phase	Difference of Magnitude	Light	O.−C. Light
0^h28	1.360	0.358	−0.002
0.74	1.180	0.422	+0.021
1.18	1.085	0.461	−0.004
1.61	0.925	0.534	−0.008
2.31	0.665	0.679	−0.001
2.80	0.485	0.801	+0.024
3.55	0.360	0.899	−0.014
4.07	0.255	0.990	−0.001
4.53	0.195	1.047	+0.004

It is not necessary to set down here the details of the numerical work by which I arrived at different sets of elements. Let it suffice to give merely the results of the successive approximations. Starting with the magnitudes in Table III, and using equations (2), (3), and (4) as indicated, the different elements are as follows:

TABLE IV

ELEMENTS

	I	II	III
κ	1.00	1.15	1.14 ±0.05
r	4.54	4.85	4.77 ±0.05
i	84°.0	82°.6	82°.3 ±0°.3
λ_1	0.049	0.038	0.038±0.007
λ	0.045	0.054	0.050±0.010
$[vv]$	0.002318	0.001810	0.001315

An inspection of the numerical solution for Elements III shows that one more approximation will reduce $[vv]$ by about 2 per cent, but I consider this refinement as unnecessary. The probable errors of κ, r, and i were computed, in the usual way, from the weights derived from the normal equations; but for the probable errors of λ and λ_1, I have assumed that the probable errors of L_m and L_M are respectively ± 0.003 and ± 0.005 light-units. In Fig. 4 the system of *Algol* as seen from the earth has been drawn from the data of Elements III.

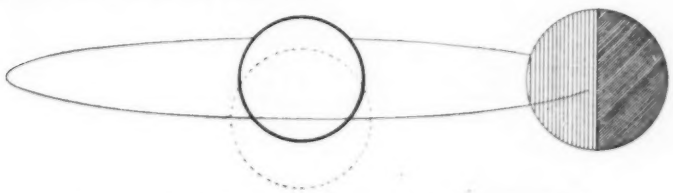

FIG. 4.—The System of *Algol* as Seen from the Earth

As these elements are considerably different from those previously derived for *Algol*, it may be of interest to note the reason for this result. My value for κ shows that the companion is about one-seventh larger in diameter than *Algol*, while previous determinations have yielded a value of about 0.75. The question at once arises as to whether there is something in the selenium photometer which gives an unusually large range of magnitude for this star. In Table V is a comparison of this range with that determined at Harvard,[1] and by Müller.[2] For the maximum with the selenium photometer I have taken an average magnitude between minima.

TABLE V

RANGE IN MAGNITUDE OF *ALGOL*

	Harvard	Müller	Selenium Photometer
	Mag.	Mag.	Mag.
Maximum....	2.1	2.43	0.15 fainter than *a Persei*
Minimum	3.2	3.55	1.37 fainter than *a Persei*
Range	1.1	1.12	1.22

There is no evidence of a large difference in scale between my results and those derived from visual observation, but in any event

[1] *Harvard Annals*, 50, 202, 1908. [2] *Op. cit.*, p. 193.

it is my opinion that the selenium photometer gives more nearly the absolute scale than can be obtained visually.

Assuming for the moment that the companion gives no light, and that the range of variation is 1.22 magnitudes, it follows that 0.675 of the bright disk is obscured at minimum, which gives 0.82 as the least possible value of κ. It is evident that if the companion is not wholly dark, or if it is not wholly projected against the bright star at principal minimum, then the radius, κ, may be considerably larger than unity.

From Elements III the adopted light-curve has been computed from (1) between minima, and from (2) during eclipse. The light L has been converted into stellar magnitudes, and as thousandths of a magnitude determined with the selenium photometer would have no meaning in either the Harvard or Potsdam systems, both the computed and observed magnitudes are still referred to *a Persei*.

TABLE VI

ADOPTED LIGHT-CURVE OF *ALGOL*

Phase	Difference of Magnitude	Phase	Difference of Magnitude
±0^{h}0	1.37	30^{h}0	0.129
±0.5	1.31	31.0	0.138
±1.0	1.15	32.0	0.152
±1.5	0.95	33.0	0.170
±2.0	0.77	34.0	0.186
±2.5	0.61	34.67	0.189
±3.0	0.47	35.0	0.188
±3.5	0.35	36.0	0.177
±4.0	0.265	37.0	0.160
±4.5	0.200	38.0	0.142
±4.90	0.174	39.0	0.131
+5.0	0.174	39.6	0.128
7.5	0.170	40.0	0.128
10.0	0.166	45.0	0.136
15.0	0.156	50.0	0.147
20.0	0.144	55.0	0.158
25.0	0.134	60.0	0.168
29.8	0.128		

This light-curve has already been shown in Fig. 3, and it should be understood that the curve there drawn has been derived entirely by computation from the elements, except that I made a graphical determination of the times of minima.

The duration of the eclipse is easily found from the equation

$$\cos \phi_1 = \frac{\sqrt{r^2-(1+\kappa)^2}}{r \sin i},$$

where ϕ_1 represents the true anomaly at first or last contact. Using Elements III, I obtain

Duration of eclipse $= 9^h.80$.

The conclusion of Müller that the principal minimum lasts 12 or 13 hours has therefore not been verified, but he was not fortunate in securing enough observations near the critical times of beginning and ending to establish this longer time. Rödiger[1] has used Müller's light-curve as a basis of deriving the elements of *Algol* with allowance for an assumed decrease in intensity of the bright disk from center to limb, but likewise his results are not confirmed. Judging from the beautiful accordance of Müller's observations near the time of greatest eclipse, it is my opinion that if he had been able to push his observations to the limit throughout the entire period of the star's variation, he would have detected the secondary minimum.

COMPARISON OF *ALGOL* WITH THE SUN

As any direct determination of the intrinsic luminosity of a star requires that its distance be known, we are fortunate in having a pretty fair determination of the parallax of *Algol*. I am indebted to Professor Comstock for a statement of his best judgment as to the parallax of this star, together with an unpublished determination by Flint at Madison. In Table VII the results by Chase and Pritchard have been increased by $0''.01$ on account of assumed parallax of the comparison stars.

TABLE VII

PARALLAX OF *ALGOL*

Observer	π	Weight
Chase.....	$+0''.05$	3
Pritchard	$+0.07$	1
Flint.....	$+0.12$	1
Adopt.....	$+0''.07$	

[1] *Untersuchungen über das Doppelsternsystem Algol*, Königsberg, 1902.

With this value of the parallax it is now easy to compute the luminosity, or total light, of *Algol* as compared with the sun. Let l be the luminosity, π the parallax, and M the stellar magnitude of the bright component of *Algol;* also let S be the stellar magnitude of the sun. From simple considerations we have

$$\log l = -2 \log \sin \pi - 0.4\,(M-S).$$

Adopting $M = 2.2$, and $S = -26.6$, there follows:

TABLE VIII

		$\odot = 1$	Stellar Magnitude
l	Total light of *Algol*	26	2.2
$lk^2\lambda$	Total light of faint hemisphere of companion	1.7	5.2
$lk^2(\lambda+\lambda_1)$	Total light of bright hemisphere of companion	3.0	4.6

The principal source of error in the comparison with the sun lies of course in the parallax. If instead of $\pi = 0''.07$ we adopt the results of Chase's heliometer measures, $\pi = 0''.05$, the computed luminosities in Table VIII will be doubled, and I am inclined to look upon the tabulated values as too small. In any case we have the result that the light from one side of the companion is nearly twice, and from the other, three times that of the sun. If the bright star could be extinguished, the companion would appear to us as varying on its own account, the range being from 4.6 to 5.2 magnitudes.

DIMENSIONS OF THE SYSTEM OF *ALGOL*

As is well known, Vogel's determination of the variable radial velocity of *Algol* led to the first approximation of the actual diameter in kilometers of any of the fixed stars; but unfortunately Vogel's numerical results are often quoted without due emphasis on his necessary assumption that the two components are of the same density. For a comparison with my light-curve there are available the results by Schlesinger and Curtiss[1] from the observations in 1906 and 1907. In the conventional notation, the elements which are applicable are as follows:

[1] *Publications of the Allegheny Observatory*, 1, 25, 1908.

TABLE IX

ALLEGHENY SPECTROSCOPIC ELEMENTS

	Schlesinger	Curtiss	Adopted
e	0.031	0.060	0.05
ω	$+44^\circ$	-3°	$+21^\circ$
$a \sin i$	1,570,000 km	1,630,000 km	1,600,000 km

There was a discrepancy between the time of light-minimum as determined photometrically and as demanded by these velocity determinations, amounting to $1\frac{1}{2}$ to 2 hours. Since my correction to the light-ephemeris in 1909–1910 amounts to $-1^h\,16^m$, a portion of this discordance is removed; but as I have learned from Director Schlesinger that the orbit of *Algol* is still being investigated at Allegheny, it seems best to wait for a comparison of spectrographic and photometric observations taken near the same epoch.

As only one spectrum was discernible on the plates, the spectroscopic elements refer to the motion of the bright component about the center of mass. Evidently, when $\omega = 0^\circ$ the distances between centers of the projected disks are equal at the two minima, and in view of the small value derived for ω, my determination of the relative brightness of the components has not been materially affected by neglecting the eccentricity. Likewise a simple calculation shows that the assumption of a circular orbit introduces no appreciable error into the computed variation of the light during the eclipses. In my judgment, it is as yet impossible to determine the eccentricity of this orbit from photometric observations, and all the numerical results which have been derived from the asymmetry of the light-curves by visual observers are spurious. Nevertheless, it may be of passing interest to compare the intervals between minima as determined graphically from my light-curve, and as computed from the Allegheny elements. The total period being $68^h.816$, I find the following times by which the secondary follows the principal minimum:

$e=0$	$34^h.41$
From light-curve	34.67
From Allegheny elements	36.46

It is apparent that the photometric result is not in agreement with the Allegheny determination of 3 years previous, but in view of the difficulty of finding the exact time of secondary minimum from a variation of only 0.06 magnitude, I attribute little significance to this fact.

Before deriving plausible values of the dimensions in the system of *Algol*, on the basis of the spectroscopic orbit, let us consider the probability of various assumptions as to the relative masses of the two bodies. Although we have many theories of stellar evolution, to my mind the most direct evidence which bears upon this question is that found by Schlesinger and Baker[1] in a comparative study of spectroscopic binaries. They have announced the rule thus far without exception, that where the spectra of both components of a close binary are visible, the brighter star is always the more massive. Therefore when the spectrum of the fainter component is not visible, the mass of the bright star is presumably much the greater. With this in mind, I have thought it best to compute the dimensions on two assumptions: first, that the two bodies are of equal density, and second, that *Algol* has twice the mass of the companion.

The sun being taken as unity, let m and m_s represent the masses, and d and d_s the densities of the two bodies, the subscript referring to the companion. Changing to conventional notation, we have $r=a+a_s$. The combined mass is given by

$$m+m_s=\frac{4}{10^{20}}\,\frac{(a+a_s)^3}{P^2},$$

where the distances are expressed in kilometers and the period in days. From $a \sin i = 1{,}600{,}000$ km and $i=82°.3$ it follows that $a=1{,}610{,}000$ km. To compute $(a+a_s)$ we have the two assumptions

(1), $d=d_s$	(2), $m=2m_s$
$a+a_s=a\left(1+\frac{1}{\kappa^3}\right)$	$a+a_s=3a$

The radius of the sun being 697,000 km, I find from simple computations the results in Table X, where each quantity is referred to the sun.

[1] *Publications of the Allegheny Observatory*, 1, 135, 1910.

TABLE X

		(1), $d=d_s$	(2), $m=2m_s$
R	Radius of *Algol*	0.81 ⊙	1.45 ⊙
R_K	Radius of companion	0.92	1.66
m	Mass of *Algol*	0.04	0.37
m_s	Mass of companion	0.06	0.18
d	Density of *Algol*	0.07)	0.12
d_s	Density of companion	0.07)	0.04
σ	Surface-intensity of *Algol*	40	12
$\sigma\lambda$	Surface-intensity of faint hemisphere of companion	2.0	0.6
$\sigma(\lambda+\lambda_1)$	Surface-intensity of bright hemisphere of companion	3.5	1.1

On the first assumption there results a great surface-intensity for a small mass, while in the second case, where the companion is supposed to be relatively rare, the masses and radiative powers are much nearer those of the sun. On the basis of these figures alone, it would seem that the second assumption is nearer the truth.

The low density of *Algol* stars was shown independently by Roberts[1] and Russell,[2] though the first determination of the density of *Algol* itself seems to be due to Mériau.[3] From my elements, I find

Mean density of the system	0.07 ⊙
Limiting density of *Algol*	0.18
Limiting density of companion	0.12

It is to be hoped that a means of detecting the spectrum of the companion may be found, which will at once give us the actual masses, diameters, and densities of the two bodies, subject only to the errors of observation. In this connection, it is barely possible that the extra sensibility of selenium at the red end of the spectrum[4] may have been a factor in the detection of the secondary minimum in the light-curve, and it would be of interest to know the spectral type of each hemisphere of the companion.

We now have numerical data on which to base a plausible explanation of the greater luminosity of the companion on the side toward the primary. Two reasons at once suggest themselves: first, that the companion reflects the light of the bright star, and

[1] *Astrophysical Journal*, 10, 308, 1899. [2] *Ibid.*, 10, 315, 1899.

[3] *Comptes Rendus*, 122, 1254, 1896.

[4] The color-sensibility curve of the selenium cell, Giltay 93, which was used in the observations of *Algol*, will be found in the *Astrophysical Journal*, 27, 185, 1908.

second, that the radiation from *Algol* is so intense that it is sufficient to heat the near hemisphere of the companion up to incandescence.

From the values of κ and r, and the solid angle at the center of *Algol* subtended by the companion, I find the fraction of the total light intercepted by the companion to be 0.015. Hence it is quite impossible that the relative light reflected from the bright side of the companion should amount to the previously computed value, $\kappa^2\lambda_1 = 0.049$. It seems, therefore, that the increase in brightness of one hemisphere must be due to the heating effect of the intense radiation emitted by *Algol*.

Consider the sphere concentric with *Algol*, and of radius equal to that of the relative orbit. The radiation received per unit surface of this sphere is $1/r^2 = 1/22.8$ times the radiation emitted per unit surface of *Algol*. This fraction represents roughly the relative radiation received by the companion. On the hypothesis of equal density, the surface-intensity of *Algol* is 40, and it would follow that the companion receives $40/22.8$, or nearly twice as much light per unit surface as is emitted by the sun. Presumably this value is too high, and it refers only to a limited portion of the spectrum. Of course the determination of the total energy of the radiation received by the companion is an entirely different problem, but in any event these figures are suggestive.

The increase of luminosity on one side of the companion, as shown by my light-curve, is a striking independent confirmation of the results of Nordmann.[1] From measures of the relative intensity of different portions of the visible spectrum, he has derived a value of the effective temperature of *Algol*, and from this he concludes that the radiation at the distance of the companion is sufficient to raise a body to incandescence. "Il est donc probable que, par le seul effet de ce rayonnement et independamment de sa chaleur propre, une partie de la surface du satellite d'*Algol* tournée vers l'étoile principale, est portée à l'incandescence." Although all determinations of stellar temperatures are as yet open to question, and the parallax of *Algol* is so small as to introduce uncertainty into the results, the main conclusion of Nordmann is well confirmed by my observations.

The question of the interaction of the radiation of two com-

[1] *Bulletin Astronomique*, 27, 145, 1910.

ponents of a close variable star has not been considered heretofore, because of the small accuracy obtainable in photometric measures. It is evident that such an effect will not be discernible where the two bodies involved are of the same order of brightness, but in many *Algol* variables where the primary is a star of great intrinsic brilliancy, and the companion relatively faint, there is every reason to suppose that future observations will demonstrate that the conditions in *Algol* are typical of this class of stellar system.

SUMMARY

1. It has been demonstrated that for bright stars the selenium photometer, attached to a 12-inch telescope, yields results which are considerably more accurate than have ever been obtained by visual or photographic methods.

2. An application of this new device to observations of *Algol* has led to the discovery that the companion, far from being dark, gives off more light than our sun, and in addition is much brighter on the side which is turned toward the primary.

3. A discussion of the photometric and other results for *Algol* gives the following principal facts concerning this system:

FROM THE LIGHT-CURVE

..	Radius of *Algol*	1.00
κ	Radius of companion	1.14 ± 0.05
r	Distance between centers	4.77 ± 0.05
i	Inclination of orbit	$82^{\circ}.3 \pm 0^{\circ}.3$
..	Surface-intensity of *Algol*	1.00
λ	Surface-intensity of faint hemisphere of companion	0.050 ± 0.010
$\lambda+\lambda_1$	Surface-intensity of bright hemisphere of companion	0.088 ± 0.012
..	Total period	$68^{h}.816$
..	Duration of eclipse	$9^{h}.80$
..	Mean density of the system	$0.07\odot$
..	Limiting density of *Algol*	$0.18\odot$
..	Limiting density of companion	$0.12\odot$

FROM $\pi = 0''.07$; SUN $= -26.6$ MAG., *Algol* $= 2.2$ MAG.

		$\odot = 1$	Stellar Magnitude
l	Total light of *Algol*	26	2.2
$l\kappa^2\lambda$	Total light of faint hemisphere of companion	1.7	5.2
$l\kappa^2(\lambda+\lambda_1)$	Total light of bright hemisphere of companion	3.0	4.6

FROM $a \sin i = 1{,}600{,}000$ km, TWO ASSUMPTIONS

		$d = d_s$	$m = 2m_s$
R	Radius of *Algol*	0.81 ⊙	1.45 ⊙
$R\kappa$	Radius of companion	0.92	1.66
m	Mass of *Algol*	0.04	0.37
m_s	Mass of companion	0.06	0.18
d	Density of *Algol*	0.07	0.12
d_s	Density of companion	0.07	0.04
σ	Surface-intensity of *Algol*	40	12
$\sigma\lambda$	Surface-intensity of faint hemisphere of companion	2.0	0.6
$\sigma(\lambda+\lambda_1)$	Surface-intensity of bright hemisphere of companion	3.5	1.1

In conclusion, I beg to acknowledge my indebtedness to Dr. F. C. Brown, in collaboration with whom many of the preliminary experiments were made; also to Dr. F. W. Reed, and Mr. Percy F. Whisler for efficient assistance in taking the observations of *Algol*. I also desire to thank the Rumford Committee of the American Academy of Arts and Sciences for successive grants of $200 and $350 which enabled me to carry on this work.

UNIVERSITY OF ILLINOIS OBSERVATORY
August 1, 1910

NOTE ADDED AUGUST 12, 1910

I have just received the list of parallaxes by Kapteyn and Weersma in *Groningen Publications No. 24*, which contains an additional determination by H. N. Russell, who finds for *Algol*, $\pi = +0''.01$. Kapteyn rejects Pritchard's result, and using only the values of Chase and Russell, he adopts $\pi = +0''.029$; also he uses -26.1 as the sun's magnitude in the derivation of luminosities. Recomputing the results of Table VIII on this basis, we have

Total light of *Algol*	240 ⊙
Total light of faint hemisphere of companion	16
Total light of bright hemisphere of companion	28

These luminosities are nearly 10 times as large as those which I adopted, and demonstrate that at the present time it is not possible to fix a reliable value for the light of *Algol*, except that we may say that the companion, even on the faint side, is much more luminous than the sun.

SECONDARY STANDARDS OF WAVE-LENGTH, INTERNATIONAL SYSTEM, IN THE ARC SPECTRUM OF IRON

ADOPTED BY THE SOLAR UNION, 1910

At the fourth meeting of the International Union for co-operation in Solar Research, held at Mount Wilson August 31, 1910, it was unanimously agreed to establish as secondary standards of wave-length in the "International System" the means of the values obtained by three independent observers of the wave-lengths of certain lines in the arc spectrum of iron. These observers were Fabry and Buisson, working at Marseilles, Eversheim at Bonn, and Pfund at Baltimore. The committee of the Union which made the report on this subject was authorized to publish these secondary standards, and the following values—together with the individual results of the different investigations—are accordingly presented. It was also voted at this meeting

Adopted Wave-Lengths	Fabry and Buisson	Eversheim	Pfund
4282.408	.407	.408	.408
4315.089	.089	.089	.089
4375.934	.935	.934	.934
4427.314	.314	.313	.316
4466.556	.554	.557	.558
4494.572	.572	.571	.572
4531.155	.155	....	.155
4547.853	.854	.853	.853
4592.658	.658	.658	.657
4602.947	.944	.948	.948
4647.439	.437	.441	.439
4691.417	....	.419	.416
4707.288	.287	.292	.286
4736.786	.785	.787	.787
4789.657	.657	.658	.657
4878.225	.226	.224	.225
4903.325	.324	.327	.324
4919.007	.006	.007	.008

Adopted Wave-Lengths	Fabry and Buisson	Eversheim	Pfund
5001.881	.880	.885	.879
5012.073	.072	.074	.072
5049.827	.827	.827	.827
5083.344	.343	.346	.343
5110.415	.415	.414	.416
5167.492	.492	.491	.492
5192.363	.362	....	.364
5232.957	.958	.958	.956
5266.569	.568	.569	.569
5371.495	.498	.493	.494
5405.780	.780	.780	.780
5434.527	.530	.524	.528
5455.614	.616	.611	.614
5497.522	.521	.523	.523
5506.784	.783	.785	.784
5569.633	.632	.636	.631
5586.772	.770	.773	.772
5615.661	.658	.662	.663
5658.836	.835	.838	.835
5763.013	.013	.013	.014
6027.059	.059	....	.059
6065.492	.493	.493	.491
6137.701	.700	....	.702
6191.568	.569	.568	.567
6230.734	.732	.736	.735
6265.145	.147	....	.143
6318.028	.029	.028	.026
6335.341	.343	.342	.337
6393.612	.612	.613	.612
6430.859	.859	.862	.855
6494.993	.994	.994	.992

of the Union that wave-lengths measured in this system should be designated in the future by using the symbol "I.A." to indicate the unit used.

Members present of the Committee on Standards of Wave-Length

H. KAYSER
CH. FABRY
J. S. AMES

MOUNT WILSON, CALIFORNIA
September 1, 1910

STANDARDS OF THIRD ORDER OF WAVE-LENGTHS ON THE INTERNATIONAL SYSTEM

By H. KAYSER

The following table contains measurements of the arc spectrum of iron photographed in the second order with an excellent Rowland concave grating of twenty-one feet radius. The plates were made within the past five years, in part by Professor Konen, and in part by Professor Eversheim, Dr. Bachem, and myself. The measurements were made by myself, with a dividing engine constructed by Wolz.

I have employed as standards the arithmetical means of the measurements of Fabry and Buisson, Eversheim, and Pfund. The portions of spectrum measured were always between three or four successive standards, so that each line is referred to as many neighboring standard lines as possible. It turned out that measurements of the same sharp line on different plates yielded differences of not more than from 0.001 to 0.002 Å, when the same standards were employed; but if different standards were employed, differences as great as about 0.006 occurred. This proves that some of the standards still contain errors of from 0.004 to 0.005 Å, and that the measurements of the best plates in the second order give a greater degree of accuracy than that of the secondary standards. Each line was measured from ten to twenty times, and from all the measurements the arithmetical mean is taken, the probable error also being given.

This method of measurement has also yielded values for the standards themselves, which I regard as somewhat more accurate than the secondary international standards, inasmuch as they are adjusted. They are in general in good accord and only in a few cases is the difference large, in the case of λ 6430 reaching the amount of 0.012 Å. It remains to be investigated whether my value is more accurate than the other.

A comparison with Rowland's solar spectrum wave-lengths shows that the differences vary irregularly between 0.15 and 0.22 Å. By substracting about 0.19 Å, all measurements on Rowland's system can be reduced to the international system with sufficient accuracy.

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
4118.553	3	1	.708	□.155				
21.807	1	1	.963	.156				
27.615	2	1	.767	.152				
32.064	6	1	.235	.171				
34.684	3	1	.840	.156	.685			
37.003	3	1	.156	.153				
43.421	4	1	.572	.151				
43.873	7	1	.038	.165				
47.674	2	1	.836	.162	.677			
54.506	2	1	.667	.161				
56.806	2	1	.970	.164				
70.906	2	1	.068	.162				
75.642	2	1	.806	.164				
84.894	2	1	.058	.164				
87.050	4	1	.204	.154				
87.807	4	1	.943	.136				
91.441	4	1	.595	.154	.441			
91.677	1	2	.843	.166				
98.307	4	2	.494	.187				
99.099	4	1	.267	.168				
4202.030	5	1	.198	.168				
10.357	2	1	.494	.137				
16.185	3	1	.351	.166				
19.365	3	1	.516	.151				
22.220	3	1	.382	.162				
27.440	2	1	.606	.166				
33.610	3	1	.772	.162	.615			
38.824	3	1	.970	.146				
47.437	3	1	.591	.154				
50.129	4	1	.287	.158				
50.789	5	1	.945	.156				
54.334	2	1	.505 *Ca*	.171				
60.484	7	1	.640	.156				
67.828	2	1	.985	.157				
71.163	6	1	.325	.162				
71.745	7R	1	.934	.189				
71.838	2	2	.045	.207				
82.404	4	1	.565	.161	.407	.408	.410	
94.124	4	1	.301	.177				
99.243	5	1	.410	.167				
4307.872	6R	2	.907 *Ca*	.135				
07.945	4R	2	.081	.136				
15.089	4	1	.262	.173	.089	.089	.092	
25.763	8R	..	.939	.176				
37.053	4	1	.216	.163				
52.740	4	1	.908	.168	.741	.741	.742	
58.500	2	1	.670	.170				
69.767	3	2	.941	.174				
75.930	4	1	.107	.177	.935	.934		
83.552	10R	1	.720	.168				
90.950	2	1	.123	.173				
4404.753	8R	1	.927	.174				
15.125	6R	1	.293	.168				

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
4422.562	3	1	.741	0.179				
27.314	4	1	.482	.168	.314	.313	.314	
30.618	3	1	.785	.167				
42.343	4	1	.510	.167				
47.723	4	1	.892	.169				
54.382	3	1	.552	.170				
59.120	3	1	.301	.181				
61.652	4	1	.818	.166				
66.553	4	1	.727	.174	.554	.557		
69.382	4	1	.545	.163				
82.163	3	1	.338	.175				
82.266	4	1	.438	.172				
89.744	3	1	.911	.167				
90.088	2	1	.253	.165				
94.571	4	1	.738	.167	.572	.571	.572	
4514.192	1	1	.358	.166				
17.530	2	1	.702	.172				
25.152	3	1	.314	.162				
28.622	4	2	.798	.176		.622		
31.159	3	1	.327	.168	.155		.154	
47.859	3	1	.024	.165	.854	.853		
56.127	3	1	.306	.179				
66.526	1	1	.693	.167				
74.729	1	1	.899	.170				
81.527	2	1	.693	.166				
87.135	1	1	.308	.173				
92.656	3	1	.840	.184	.658	.658		
98.136	2	3	.303	.167				
4602.004	2	2	.183	.179				
02.945	4	2	.126	.181	.944	.948		
11.295	3	1	.469	.174				
13.225	2	3	.386	.161				
19.295	3	3	.468	.173				
25.064	4	1	.227	.163				
32.917	3	1	.100	.183				
37.517	3	1	.685	.168				
38.019	3	1	.193	.174				
47.436	3	1	.617	.181	.437	.441		
54.495	3	1	.672	.177				
54.639	3	1	.800	.161				
67.457	3	2	.626	.169				
68.148	3	1	.331	.183				
73.167	3	2	.347	.180				
78.854	4	2	.027	.173	.855			
83.580	2	1	.745	.165				
91.415	3	1	.602	.187				
4707.289	3	1	.457	.168	.287	.292		
07.492	1	2	(.672)	.180				
10.280	2	1	.471	.191				
20.999	1	1	.179	.180				
27.438	1	1	.582	.144				
33.592	2	1	.779	.187				
36.787	4	2	.963	.176	.785	.787		

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
4741.530	2	1	.718	0.188				
45.802	2	2	.992	.190				
54.049	..	1	.225	.176	.046	.049		*Mn*
57.576	2	1	.771	.195				
62.371	..	1	.567	.186				*Mn*
66.422	..	2	.621	.199				*Mn*
72.816	2	2	.007	.191				
79.439	1	2	.634	.195				
83.436	..	2	.613	.177				*Mn*
86.811	2	2	.003	.192				
89.651	2	1	.849	.198	.657	.658		
98.278	2	3	.453	.175				
4800.653	2	2	.842	.189				
02.886	1	2	.072	.186				
07.735	1	3	.900	.165				
23.523	..	1	.697	.174	.521	.523		*Mn*
32.734	2	1	.905	.171				
39.546	2	1	.735	.189				
43.161	2	1	.336	.175				
45.652	1	1	.843	.191				
55.693	1	1	.859	.166				
59.754	4	1	.928	.174	.756	.758	.755	
63.664	1	1	.833	.169				
71.333	5	1	.512	.179				
72.155	4	1	.332	.177				
78.229	4	1	.407	.178	.226	.224	.223	
81.724	2	2	.904	.180				
82.166	2	2	.336	.170				
85.445	3	1	.620	.175				
90.771	5	1	.948	.177				
91.510	6	1	.683	.173				
4903.323	4	1	.502	.179	.324	.327	.325	
10.031	2	1	.198	.167				
19.014	5	1	.174	.160	.006	.007	.005	
20.523	7	1	.685	.162				
24.777	2	1	.956	.179				
30.348	1	1	.486	.138				
38.195	2	1	.350	.155				
38.830	3	1	.997	.167				
46.408	2	1	.568	.160				
50.140	1	2	.291	.151				
57.303	4	1	.480	.177				
57.609	5	1	.785	.176				
66.105	3	1	.270	.165	.104	.105		
70.506	1	1	.671	.165				
78.620	2	1	.785	.165				
82.529	3	1	.682	.153				
85.274	3	1	.432	.158				
85.569	3	1	.730	.161				
94.139	2	1	.316	.177				
5001.884	3	1	.044	.160	.880	.885	.883	
05.732	3	1	.896	.164				
06.136	4	1	.306	.170				

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
5012.070	3	2	.252	0.182	.072	.074		
14.958	3	2	.123	.165				
22.258	2	1	.414	.156				
28.136	2	2	.308	.172				
39.264	2	1	.428	.164				
41.079	2	1	.255	.176				
41.758	2	2	.936	.178				
49.825	3	3	.008	.183	.827	.827		
51.640	4	1	.825	.185				
68.786	3	1	.944	.158				
74.740	3	1	.932	.192				
79.231	2	1	.409	.178				
79.751	2	1	.921	.170				
83.346	2	1	.518	.172	.343	.346		
90.782	2	1	.954	.172				
96.999	2	1	.175	.176				
5107.462	3	1	.619	.157				
07.650	3	1	.823	.173				
10.412	3	1	.574	.162	.415	.414		
23.729	2	1	.899	.170				
27.366	2	1	.533	.167				
31.482	3	1	.642	.160				
33.675	1	1	.870	.195				
39.268	3	1	.427	.159				
39.482	4	1	.644	.162				
42.934	2	1	.111	.177				
50.841	2	1	.020	.179				
51.915	2	2	.087	.172				
62.310	3u	2	.449	.139				
67.488	4	1	.678	.190	.492	.491	.493	
74.184	1	1	....	....				
87.928	1	1	.079	.151				
91.477	3	1	.629	.152		.473		
92.362	4	1	.523	.161	.362			
94.946	3	1	.941	.195				
98.717	2	1	.888	.171				
5202.341	2	1	.516	.175				
08.619	1	1	.776	.157				
15.193	2	1	.353	.160				
16.275	2	1	.437	.162				
17.403	2	1	.552	.149				
26.874	3	1	.043	.169				
27.185	4	1	.362	.177				
29.843	2	1	.030	.187				
32.960	5	1	.122	.162	.958	.958	.956	
42.495	2	1	.658	.163				
54.959	1	1	.121	.162				
59.740	1	1	(.912)	.172				
63.319	2	1	.486	.167				
66.574	4	1	.738	.164	.568	.569		
69.535	6	1	.723	.188				
70.354	4	2	.558	.204				
73.176	2	1	.339	.163				

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
5273.378	2	1	.558	0.180				
81.806	4	1	.971	.165				
83.635	4	1	.802	.167				
5302.314	3	1	.480	.166	.316	.316		
07.364	2	1	.541	.177				
24.197	5	2	.373	.176	.196	.196		
28.040	5	1	.236	.176				
28.532	4	1	.696 .747	.189				
32.904	3	1	.089	.185				
39.947	3	1	.121	.174				
41.027	4	1	.213	.186				
53.390	2	2	.571	.181				
64.862	3u	1	.069	.207				
65.404	2	1	.596	.192				
71.490	3	2	.734	.244	.498	.493		
79.583	2	2	.775	.192				
83.360	5	2u	.578	.218				
89.482	2	3u	.683	.201				
93.186	4	1	.375	.189				
5400.510	2	2	.711	.201				
05.778	5	1	.989	.211	.780	.780		
10.902	3	1	.124	.222				
15.184	4u	1	.416	.232				
24.050	5u	2	.290	.240				
29.701	5	1	.911	.211				
34.527	4	1	.740	.213	.530	.524	.529	
45.037	2	1	.259	.222				
46.918	4	1	.130	.212				
55.439	2	1	.671	.232				
55.616	3	1	.834	.218	.616	.611	.615	
62.964	2	1	.174	.210				
63.272	3	1	.494	.222				
66.424	2	1	.609	.185				
73.910	2	1	.113	.203				
76.294	2	1	.500	.206				
76.581	3	1	.778	.197				
87.768	2	2	.959	.191				
97.521	4	1	.735	.214				
5501.469	4	1	.683	.214				
06.783	4	1	.000	.217				
09.529	1	1	(.756)	.227				
25.559	2	1	.765	.206				
35.420	3	1	.644	.224	.418			
43.184	2	1	.414	.230				
43.944	2	2	.157	.213				
54.878	2	1	.122	.244				
63.608	3	1	.824	.216				
65.685	3	2	.931	.246				
69.630	4	1	.848	.218	.632	.636		
72.858	5	1	.075	.217				
76.102	3	1	.320	.218				
86.774	7	1	.991	.217	.770	.773	.770	

Å	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
5598.292	2	2	.524	0.232				
5602.962	3	2	.186	.224				
15.667	8	1	.877	.210	.658	.662	.657	
24.562	4	1	.769	.207				
38.289	2u	2	.488	.199				
48.323	2	1	(.503)	.180				
58.846	4	1	.052	.206	.835	.838		
62.537	3	2	.744	.207				
75.305	2u	2	....	....				
82.222	1u	2	.427	.205				*Ni*
92.220	2	1	....	....				
5701.561	3	1	.772	.211				
09.394	4	1	.601	.207	.396			
09.584	..	1	.775	.191				*Ni*
15.111	3	1	.308	.197				
17.863	2	1	.055	.192				
31.774	3	3	.984	.210				
48.381	..	3	.576	.195				*Ni*
53.142	3	1	.344	.202				
54.687	..	1	.881	.194				*Ni*
63.014	4	1	.218	.204	.013	.013		
82.156	1	3	....	....				
91.044	1	3	.243	.199				
5805.212	..	3	.441	.228	.211			*Ni*
08.750	1u	2	....	....				
17.044	1u	3	....	....				
30.562	1u	3	....	....				
32.783	1u	3	....	....				
42.461	1u	3	....	....				
46.996	..	3	.221	.225				*Ni*
54.420	1u	3	....	....				
57.753	..	3	.976	.223	.760	.759		*Ni*
62.322	4	2	.582	.260				
72.897	1	3	....	....				
77.959	1	3	....	....				
83.822	2	3	.028	.206				
92.897	4	3	.097	.200	.882	.881		*Ni*
5905.661	3	3	.895	.234				
14.142	4u	3	.335	.193				
16.241	2	3	.475	.234				
30.152	3u	3	.406	.254				
34.668	3	3	.881	.213	.683			
52.739	3	3	.943	.204	.739			
56.695	2	3	.923	.228				
75.354	2	2	.575	.221				
76.800	2	2	.007	.207				
83.709	3	2	.908	.199				
84.795	4	1	.040	.245				
87.045	3	1	.290	.245				
6003.029	3	1	.239	.210	.039			
07.953	3	1	.186	.233				
08.580	4	1	.785	.205				
13.516		1	.715	.199				*Mn*

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
6016.666	..	1	.861	0.195				*Mn*
20.167	4u	1	.401	.234				
21.821	..	1	.016	.195				*Mn*
24.047	5	1	.281	.234				
27.062	4	1	.274	.212	.059			
42.092	2	1	.315	.223				
55.992	3	1	.227	.235				
65.489	5	2	.709	.220	.493	.493		
78.476	3	2	.710	.234				
89.581	2	2	.787	.206				
90.229	..	2	.429	.200				
96.708	..	3	.880	.172				
6102.182	4	3	.392	.210				
03.192	3	1u	.400	.208				
27.927	2	1	.124	.197				
36.628	5	1	.829	.201				
37.012	1	1	.210	.198				
37.703	5	1	.915	.212	.700			
47.844	2	1	.040	.196				
51.633	3	1	.834	.201				
57.740	4	1	.945	.205				
65.387	1	2	.577	.190				
73.350	3	1	.553	.203				
80.230	3	1	.420	.190				
91.568	5	1	.779	.211	.569	.568	.566	
6200.330	5	1	.527	.197				
13.438	4	1	.644	.206				
15.153	3	1	.360	.207				
19.289	4	1	.494	.205				
29.251	2	2	.437	.186				
30.734	6	1	.943	.209	.732	.736	.729	
32.669	3	1	.856	.187				
40.666	2	1	.863	.197				
46.345	4	1	.535	.190				
52.567	5	1	.773	.206				
54.269	3	1	.456	.187				
56.377	3	1	.572	.195				
65.145	3	1	.348	.203	.147			
70.245	1u	3	.442	.197				
80.631	2u	3	.833	.202				
97.804	3	1	.007	.203				
6301.528	4	1	.718	.190				
02.522	3	1	.709	.187				
15.323	2	1	.517	.184				
15.822	2	1	.028	.206				
18.031	5	1	.239	.208	.029	.028		
22.198	2	1	.907	.209				
35.339	5	1	.554	.215	.343	.342		
36.850	5	1	.048	.198				
44.164	1u	2	.371	.207				
55.045	2	1	.246	.201				
58.693	2u	2	.898	.205				
75.206	1	3	(.455)	.249				

λ	Intensity	Probable Error	Rowland	Difference	Fabry and Buisson	Evers-heim	Pfund	
6376.198	1	2	(.385)	.187				
80.754	2	1	.958	.204				
93.612	6	1	.820	.208	.612	.613	.609	
6400.036	8	3	.217	.181				
08.045	3	2	.233	.188				
11.673	4	1	.865	.192				
21.354	3	1	.570	.216				
30.848	4	3	.066	.217	.859	.862		
62.734	2	1	.965	.231				
72.631	1	1	(.823)	.192				
81.896	2	2	(.098)	.202				
94.994	7	..	.213	.219	.994	.994	.990	

The observations will soon be published *in extenso*. I intend to continue the measurements as far as λ 7900.

September 1910

ON THE APPARENT PERIODICITY IN THE SPACING OF THE SATELLITES OF SOME OF THE MERCURY LINES

By G. F. HULL

During the year 1905–1906 the writer, while working in the Cavendish Laboratory in Cambridge, observed that the satellites of the mercury lines λ 5461, 4359, and 4047 differed in their positions from one another by quantities which had a rather large common divisor. The chief problem[1] in hand at that time, however, was not concerned with this observation, so the records were set aside for later consideration. It was expected too that an instrument of greater resolving power could be obtained by means of which the observations could be subjected to a severer test. In the meantime, other observers have published their results, so that the hypothesis that there is a periodicity in the spacing of the satellites of the mercury lines may now be examined.

Periodicity in the spacing of so-called satellites may be due to two causes. Faulty construction of the analyzing apparatus, whether interferometer, grating, single or multiple plate echelon, may produce "ghosts" of the main line which would probably be spaced in some kind of periodic order. In this case, there might be a variation in the intensities of the "satellites" which would suggest the cause of the phenomenon. To this class some of the satellites of the mercury lines in the early observations of Lummer and Gehrcke belong.[2] That there were a number of "ghosts" in those apparent satellites has since been shown by Gehrcke and v. Baeyer, using crossed echelon plates. The other possible cause of the periodicity in spacing of the satellites would lie in the structure of the atom or molecule. Let us assume that the atom consists of a sphere of positive electricity with electrons arranged in

[1] G. F. Hull, "On the Influence of Electric Fields on Spectral Lines," *Astrophysical Journal*, **25**, 1, 1907.

[2] *Annalen der Physik*, **10**, 457, 1903.

[3] *Ibid.*, **20**, 269, 1906.

concentric circles. Given the number, p, of the electrons at the center, the number on a ring required for equilibrium would lie between certain limits $n \pm e$. Thus we might have atoms with p electrons at the center and $n \pm 1$, $n \pm 2$, $n \pm 3$ electrons on a ring. Professor Thomson finds that when n is large, the frequency of the vibration arising from a tangential displacement of an electron takes the form

$$q^2 = A\left(1 - \tfrac{1}{2}\frac{S^2\pi^2}{n^2}\right)$$

where S has the values 1, 2, 3, etc.

And for a radial displacement

$$q^2 = B + C\left(1 - \tfrac{1}{2}\frac{K^2\pi^2}{n^2}\right)$$

where B has as a factor p, the number of electrons at the center. Giving to K or S the integral values 1, 2, 3, etc., the frequencies would produce a band spectrum according to Deslandres' law. If we were to assume that the number of electrons in a ring varied slightly for different atoms, we would get variations in q which might account for the phenomenon of satellites. Our theory of the Zeeman effect is not complete enough to lead us to predict whether these different vibrations would be influenced by a magnetic field all in the same way. Consequently, we are not forced to reject this explanation of the origin of the satellites on account of the fact that they do not exhibit the same Zeeman effect.

It must be noted that n would have to be very large in order to account for the small variations in frequency which we find to exist. At best, we can only consider this theory as pointing to a possible cause for the existence of satellites.

In the data below are included measurements made by various observers, viz.: O. v. Baeyer,[1] L. Janicki,[2] Gehrcke and v. Baeyer[3] (the mean of three sets), Gale and Lemon.[4] The numbers +228, −122, etc., are the differences in thousandths of an Ångström unit between the satellites and the main line.

[1] *Physikalische Zeitschrift*, **9**, 831, 1908.

[2] *Annalen der Physik*, **19**, 36, 1906; **29**, 823, 1908.

[3] *Ibid.*, **20**, 278, 1906.

[4] *Astrophysical Journal*, **31**, 78, 1910.

5790.000 v. B. G. & L. −8

+228 = +45×5	+3	
+133 = +45×3	−2	
−122 = −60×2	−2	
−180 = −60×3	0	

+229	+221	= 32×7	− 3
+135	+127	= 32×4	− 1
−119	−127	= −32×4	+ 1
−181	−189	= −32×6	+ 3
−930 ?	−938 ?	= −32×2 9−10	?
−988	−996	= −32×3 1− 4	?

5769.000 v. B. +2 J. −14

+44	+46			
−48	−46			
+48	+34	= +32×1	+2	
−52	− 66	= −32×2	−2	
−114	−128	= −32×4	+0	

5461.000 J. G. & L. +6

+133 =	44×3	+1		+217	+223 =	45×5	−2
+ 88 =	44×2	+0		+130	+136 =	45×3	+1
− 66 =	−33×2	+0		+ 83	+ 89 =	45×2	−1
− 99 =	−33×3	+0		− 54	− 48 =	−45×1	−3
−232 =	−33×7	−1		− 94	− 88 =	−45×2	+2
				−233	−227 =	−45×5	−2

4358.000 G. & L. −40 +2

+194	+154 =	77×2	+0		196 =	11× 18	− 2
+118	+ 78 =	77×1	+1		120 =	11× 11	− 1
+ 40	0				42 =	11× 4	− 2
− 88	−128 =	−65×2	+2		− 86 =	−11× 8	+ 2
−155	−195 =	−65×3	+0		−153 =	−11×15	+12

4047.000 G. & v. B. −11

−115	−126 =	−64×2	+2
− 53	− 64 =	−64×1	+0
+ 75	+ 64 =	+64×1	+0
+138	+127 =	+64×2	−1

In v. Baeyer's measurements for λ 5790 two divisors have been used, 45 on one side and 60 on the other. Evidently 15 might have been used on both sides, but so small a divisor has no significance unless the remainders are small by comparison. A similar observation may be made concerning Janicki's values for λ 5461 and concerning Gale and Lemon's values for λ 4358. The small divisor 11 has no significance. In Gale and Lemon's values for λ 5790, the lines at 930 and 988 probably ought to be discarded. They are not obtained by other observers; moreover, they are so far away from the main line that they may be regarded —if they belong to the mercury spectrum—as separate lines.

The data above given, with no pretension of completeness, show how divisors may be found for the distances between the satellites and the main line. The question arises, To what extent

is this a matter of chance? In other words, given six numbers, as in Gale and Lemon's values for λ 5461, between +250 and −250, what is the probability that an integer can be found which when added to all the numbers will allow the numbers so obtained to be divided by an integer of the order of 50 so that the remainders will be less than 10 per cent of the divisor? The rather tedious arithmetical solution shows that the upper limit to the resulting probability is so small that we must regard whatever periodicity appears in the data in the table as having an existence apart from the laws of chance.

It appears, then, that for some of the mercury lines the satellites measured by some observers show periodicity not to be accounted for as accidental. Thus Gale and Lemon's values for λ 5790 (leaving out the two satellites already discussed), Janicki's values for λ 5769, Gale and Lemon's values for λ 5461, and Gehrcke and v. Baeyer's values for λ 4047 show periodicity. V. Baeyer's values for λ 5790, Janicki's for λ 5461, and Gale and Lemon's for λ 4358 show a periodicity for satellites on one side of the main line and another periodicity for satellites on the other side.

The interpretation to be given to the quantity which has been added or subtracted from the measurements is that the main line, through unsymmetrical broadening or for some other reason, has been shifted one way or the other by the amount added or subtracted. In the case or λ 4358, however, this amount shifts the zero point of reference to the satellite +40.

An inspection of the results obtained by various observers shows that the structure of the mercury lines must depend on conditions of the source not fully known. Not only does the number of the satellites vary, but the positions of those evidently the same differ by quantities larger than the experimental errors. It is evident that if any complete law is to be found for the structure of the lines of an element, not only must we examine the lines of various elements, but we must also be able to control the physical condition determining the radiation.

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PROBABLE ERRORS OF RADIAL VELOCITY DETERMINATIONS

By J. S. PLASKETT

The magnitude of the probable errors attending the spectrographic determination of stellar radial velocities has always been with me a question of much interest, and considerable work[1] along the line of the dependence of probable error upon the width of the slit employed has already been accomplished. It is proposed in this paper to give a general discussion of the probable errors of radial velocities as affected by changes in the dispersion of the instrument, and in the type of stellar spectrum observed.

It may not be amiss to point out that in measuring stellar spectra, as in practically all scientific measurements, we have two classes of errors to deal with or guard against: first, the accidental errors of setting upon the lines of the spectra due partly to imperfect definition, and partly to the unavoidable differences in successive settings which are always present even with the most careful observers; second, the systematic errors, due in this case generally to instrumental conditions which give rise to spurious relative displacements of stellar and comparison lines. Among such conditions may be cited flexure or changes of temperature of the spectrograph, non-uniform illumination of the collimator, prisms, and camera by the light from star or spark, faulty focal adjustments of collimator or camera objectives, and so on. The former can readily be evaluated from the measures of the plates themselves, but no evidence of the latter appears in such measurements, and its magnitude can only be determined from the comparison of a number of plates of the same star. In the latter case, however, the errors so obtained will not be entirely systematic but will be affected by the accidental errors present in the measured velocities. In the discussion to follow, relative measures of the accidental errors are given by the probable errors of single lines or regions on a plate, while

[1] *Astrophysical Journal*, **28**, 259, 1908; and *Trans. Roy. Soc. Can.*, **3**, 209, 1909.

for the systematic effect the probable error of a single plate, obtained by the discussion of several plates of the same star, is probably the best that can be done, although, as stated above, such result has also included in it the effect of the accidental errors.

In considering the effect of change of dispersion one would naturally expect to find the actual linear errors of the same magnitude for all dispersions and, as errors are always expressed in velocity values, the probable errors in kilometers per second would hence be inversely proportional to the linear dispersion. That is to say, as we have at the Dominion Observatory three different dispersions 10.1, 20.2, and 33.4 tenth-meters per millimeter at $H\gamma$, practically as 3, $1\frac{1}{2}$, and 1, we should expect to find the probable errors in kilometers per second inversely proportional to these latter numbers, or as 1, 2, and 3.

Most of the radial velocities at the Dominion Observatory have been obtained from spectra made with the lowest dispersion, a single-prism spectrograph, on spectroscopic binary stars of early type, in which the spectral lines have generally been broad and diffuse. The probable errors of the velocity determinations of single plates have been consequently high, so high as to lead to the belief that the relation above expressed was not the true one but that the probable errors increased more rapidly than the dispersion diminished.

It seemed therefore worth while to make a definite test of the matter especially as, although considerable data as to the probable errors of high-dispersion star spectrographs are available, there is so far as I know not much published information in regard to the probable errors of one-prism instruments. In order to avoid, so far as possible, any effect due to diffuseness of the spectral lines, it was decided to use spectra of the solar type for the comparison. Further, to eliminate difficulties of identification of wave-lengths in the blends of lines always present in low-dispersion second-type spectra, it was essential to measure the plates by the spectro-comparator, an instrument in which the actual displacement of the star lines due to velocity is compared with those in a standard plate of the sun whose velocity is known. By this method no knowledge of wave-length is necessary and errors due to the loss

of purity inherent with small dispersion cannot affect the measurements.

The brightest solar-type star, *Arcturus*, was selected as a test object for the obvious reason that only short exposures would be required. To produce spectra of good quality for measurement on *Arcturus* exposures are necessary of about ten minutes with the three-prism long-focus spectrograph designated as III L, 10.1 Ångströms per mm, four to five minutes with the three-prism short-focus designated as III R, 20.2 Å per mm, and one and a half minutes with the single-prism spectrograph designated as I, 33.4 Å per mm. at $H\gamma$, and consequently the plates required may be quickly made. Fortunately when this investigation was begun we had already obtained nearly thirty plates with III L for another purpose and only plates with III R and I were required. Eleven were made with III R and about fifty with I.

Of these plates 24 of III L, 11 of III R, and 38 of I were measured by myself on the spectro-comparator and from these measures the results to be discussed were derived. In order to explain clearly how the probable errors were obtained it is necessary to describe briefly the comparator and the method of measurement. In the first place a standard spectrum of the sun is obtained by the same spectrograph, and this plate has impressed upon it one on each side of the solar spectrum a strip of the same comparison as in the star spectrum. This standard sun spectrum and the star spectrum are viewed by a special double objective, single ocular microscope, with a Lummer Brodhun cube in the ocular, which serves to superpose the two spectra so that a narrow strip of star spectrum is seen between and touching two strips of sun spectrum, while on each side a narrow strip of the star comparison lies between and touching strips of sun comparison. The standard sun spectrum is moved by a micrometer screw until the corresponding lines of the star and sun spectra are in exact coincidence and then again moved until the comparison lines of the two spectra are coincident. The difference in the micrometer readings evidently gives us the displacement, due to radial velocity, of the star lines with respect to the sun lines, which on multiplication by a constant gives, after adding

with the proper sign the known velocity of the sun, the velocity of the star with reference to the observer.

The measures were made as usual on the spectra comparator by determining the coincidences at a number of chosen regions, marked by dots on the sun spectrum, which correspond, in a sense, to the lines in an early-type spectrum, on which the cross wire is set. The accidental errors can thus evidently be determined from the probable error of the determination of the points of coincidence in these regions. After the spectra have been measured with the violet end to the left, for example, they are reversed on the comparator, and the same regions remeasured. The differences of displacement, corrected for a systematic effect due to reversal, are evidently wholly due to accidental errors of setting, and from these the probable error of a region is readily obtained in the well-known way, giving a measure of the purely accidental error.

In addition to the purely accidental errors of setting are others also of an accidental character due to irregular arrangement of the silver grains or distortion of the film, to the forming of the coincidences to one side or other of the dot or center of the region and consequent incorrect value of the velocity-constant by which the displacement is multiplied, and to numerous other causes. A measure of the total accidental error is evidently obtained by computing the velocities separately for the mean of the two measures red right and red left of each region, by obtaining the residuals from the mean velocity of the plate, and the probable error in the usual way. This probable error should be and is, as is seen below, somewhat greater than the purely accidental error of setting.

The systematic errors due to instrumental peculiarities can only be obtained from the discussion of plates of the same star in sufficiently large numbers to insure that the systematic displacements for this particular spectrograph become accidental in character. It is possible, however, that there may be slight constant systematic differences in the values given by different spectrographs. This is shown in the results below, but is likely due to accidental errors of the fundamental spectrum. We have, in the three dispersions, plates to the number of 24, 11, and 38. By treating the resid-

uals from the mean velocities, we can obtain the probable error of a single plate in which we get a good relative idea of the systematic errors, although as before stated we have the accidental errors of measurement included. This certainly gives us an accurate idea as to the total error involved, which in the ultimate analysis is what we wish to know.

The following tables contain a summary of the measures in the three dispersions.

SPECTROGRAPH III L. 10.1 per mm

Plate No.	No. of Regions	Velocity	Probable Errors of Single Regions			Residual
			Errors of Setting		Total Accid'l Errors	
		km	Rev.	km	km	km
1426	11	−5.74	±0.0015	±0.59	±0.82	0.23
1455	11	4.97	.0008	.32	.59	0.54
1456	16	5.64	.0009	.35	.52	0.13
1514	12	5.91	.0017	.66	.93	0.40
1515	13	5.78	.0008	.31	.57	0.27
1529	10	6.14	.0010	.41	.65	0.63
1585	13	5.59	.0020	.78	.71	0.08
1586	10	5.94	.0012	.49	.61	0.43
1588	11	6.93	.0013	.53	.62	1.42
1595	12	4.14	.0009	.37	.53	1.37
1596	13	5.43	.0010	.39	.69	0.08
1597	13	6.12	.0011	.43	.66	0.71
1606	12	5.42	.0009	.37	.61	0.09
1615	12	6.24	.0011	.42	.74	0.73
1616	12	6.86	.0009	.37	.76	1.35
1619	16	5.14	.0011	.40	.32	0.37
1620	14	5.80	.0011	.43	.64	0.29
1622	15	4.58	.0015	.57	.61	0.93
1635	14	3.97	.0014	.53	.48	1.54
1645	16	5.00	.0015	.54	.64	0.51
1662	12	4.98	.0011	.45	.72	0.53
1671	16	6.10	.0009	.32	.61	0.59
1672	14	5.18	.0013	.51	.45	0.33
1709	15	4.62	.0009	.35	.74	0.89
Means	13	−5.51	±0.00117	±0.453	±0.628	

Average probable error of setting single region = ±0.45 km.
Average probable total accidental error single region = ±0.63 km.
Average probable total accidental error single plate = ±0.17 km.
Total probable error (accidental+systematic) single plate = ±0.50 km.

SPECTROGRAPH III R. 20.2 t.-m. per mm

Plate No.	No. of Regions	Velocity	Probable Errors of Single Regions			Residual
			Errors of Setting		Total Accidental Errors	
		km	Rev.	km	km	km
3288.....	10	−6.64	±0.0011	±0.80	±1.22	2.09
3289.....	9	4.94	.0009	.67	1.04	0.39
3290.....	9	5.58	.0009	.67	0.83	1.03
3291.....	10	4.83	.0013	.95	1.18	0.28
3311.....	11	3.27	.0011	.78	1.02	1.28
3312.....	10	4.16	.0013	.95	0.64	0.39
3313.....	10	3.47	.0013	.95	1.25	1.08
3314.....	10	5.78	.0010	.73	0.87	1.23
3315.....	10	3.33	.0006	.44	1.11	1.22
3316.....	10	4.52	.0008	.58	0.88	0.03
3317.....	10	3.54	.0012	.88	0.99	1.01
Means	10	−4.55	±0.00105	±0.75	±1.00	

Average probable error of setting single region = ±0.75 km.
Average probable total accidental error single region = ±1.00 km.
Average probable total accidental error single plate = ±0.32 km.
Total probable error (accidental+systematic) single plate = ±0.75 km.

SPECTROGRAPH I. 33.4 Ångströms per mm

Plate No.	No. of Regions	Velocity	Probable Errors of Single Regions			Residual
			Errors of Setting		Total Accidental Errors	
		km	Rev.	km	km	km
3126.....	6	−6.14	±0.0019	±2.25	±2.49	0.13
3127.....	8	7.87	.0009	1.02	1.25	1.86
3128.....	9	5.49	.0007	0.78	1.37	0.52
3129.....	7	6.58	.0016	1.86	1.02	1.57
3130.....	7	5.42	.0012	1.39	1.11	0.59
3146.....	8	8.54	.0008	0.90	1.57	2.53
3147*a*.....	9	6.42	.0014	1.57	1.89	0.41
3147*b*.....	9	5.49	.0011	1.23	1.39	0.52
3147*c*.....	9	5.43	.0008	0.90	1.30	0.58
3148*a*.....	11	7.56	.0007	0.76	1.52	1.55
3148*b*.....	11	6.84	.0010	1.08	1.21	0.83
3148*c*.....	11	6.19	.0006	0.65	1.80	0.18
3149*a*.....	9	5.49	.0008	0.90	1.83	0.52
3149*b*.....	9	4.44	.0011	1.23	1.31	1.57
3149*c*.....	9	4.87	.0012	1.34	1.79	1.14
3150*a*.....	9	4.44	.0007	0.78	1.25	1.57
3150*b*.....	9	4.44	.0011	1.23	1.87	1.57
3150*c*.....	9	5.99	.0007	0.78	1.45	0.02

SPECTROGRAPH I. 33.4 Ångströms per mm—*Continued*

Plat No.	No. of Regions	Velocity	Probable Errors of Single Regions			Residual
			Errors of Setting		Total Accidental Errors	
		km	Rev.	km	km	km
3238*b*	9	5.51	.0008	0.90	1.16	0.50
3238*c*	9	6.20	.0009	1.01	1.61	0.19
3239*a*	9	5.58	.0009	1.01	1.00	0.43
3239*b*	9	5.64	.0008	0.90	1.68	0.37
3239*c*	9	5.64	.0005	0.56	1.15	0.37
3240*a*	9	4.34	.0012	1.34	1.35	1.67
3240*b*	9	5.08	.0010	1.12	0.93	0.93
3240*c*	9	5.51	.0012	1.34	1.79	0.50
3272*a*	9	6.21	.0011	1.23	0.91	0.20
3272*b*	9	6.98	.0008	0.90	1.64	0.97
3272*c*	9	5.96	.0006	0.67	1.17	0.05
3273*a*	9	6.83	.0011	1.23	1.63	0.82
3273*b*	9	7.76	.0011	1.23	1.39	1.75
3273*c*	9	5.22	.0010	1.12	1.80	0.79
3274*a*	9	5.22	.0009	1.01	1.06	0.79
3274*b*	9	6.83	.0008	0.90	1.45	0.82
3274*c*	9	6.15	.0011	1.23	0.86	0.14
3275*a*	9	5.90	.0013	1.46	1.36	0.21
3280*c*	9	8.07	.0009	1.01	1.71	2.06
3281*c*	9	4.23	.0005	0.56	0.69	1.78
Means	8.84	−6.01	±0.00097	±1.09	±1.41	

Average probable error of setting single region = ±1.09 km.
Average probable total accidental error single region = ±1.41 km.
Average probable total accidental error single plate = ±0.47 km.
Total probable error (accidental+systematic) single plate = ±0.70 km.

Collecting in the following table the final values, we obtain the relative errors for the three dispersions.

Spectrograph	Scale Å per mm	No. of Plates	Mean Velocity	Errors of Single Region		Errors of Single Plates		
				Errors of Setting	Total Accidental Errors	Total Errors, Accidental and Systematic	Accidental	Systematic
III L	10.1	24	−5.51	±0.45	±0.63	±0.50	±0.17	±0.47
III R	20.2	11	−4.55	0.75	1.00	0.75	0.32	0.68
I	33.4	38	−6.01	1.09	1.41	0.70	0.47	0.52

We have in the first column the spectrograph employed; in the second the dispersion in Ångströms per millimeter at Hγ; in the

third the number of plates measured; and in the fourth the mean velocity of these plates.

The fifth column contains the average probable error of the estimation of the coincidences in a single region, while the sixth contains the average total accidental error as obtained from the final kilometer values of the displacement for each region.

Under the heading of Errors of Single Plates we have in the seventh column the total error obtained from the mean velocities of all the plates, and in the eighth the accidental error obtained by dividing the total accidental error of a single region by the square root of the number of regions, while the ninth column is obtained from the two preceding columns by taking the square root of the difference of their squares.

It is difficult to account for the results obtained, especially in the errors of single plates, for, as before stated, one would expect the kilometer values of the probable errors to be inversely proportional to the linear dispersion. This is approximately true so far as the errors of single regions are concerned, and the discrepancy can be satisfactorily explained by the greater ease and accuracy in the determination of coincidences in the single-prism spectrograph owing to the decidedly smaller curvature of the spectral lines. But when we come to the total errors of a single plate as determined from the measured velocities of the plates, we find the errors instead of being in the ratio of 1, 2, and 3, as we should expect, areas 1, $1\frac{1}{2}$, and $1\frac{1}{2}$ approximately.

So far as III L and III R are concerned, it must be remembered that, although the linear dispersions are as 1 to 2, the angular dispersions and the resolving powers are equal and the decrease of the ratio from 1:2 to $1:1\frac{1}{2}$ can thus be accounted for. In the single-prism spectrograph, however, the linear dispersion, angular dispersion, resolving power, and purity of spectrum are practically only one-third of those of III L, and the errors should be three times as great. Instead of that the total error of a plate, which is of course the most important quantity to be determined, is only 40 per cent greater with the lower dispersion and an examination of the last two columns shows that the systematic (due to changing instrumental factors) part of this error is nearly the same in III L

and I, while the accidental part is nearly in proportion to the dispersions.

This would seem to indicate either that the single-prism spectrograph is less likely to give systematic displacements of the spectrum lines than III L or that the kilometer rather than the linear value of the systematic displacements remains constant. So far as the first supposition is concerned, although the single-prism instrument[1] is undoubtedly less affected by flexure than III L and is probably better controlled and regulated as regards temperature, these two factors will not have much influence in the short exposures required on *Arcturus*. There is no ground for supposing on the other hand that kilometer rather than linear values of the systematic displacement should remain constant except in the case of displacements due to temperature changes in the prisms.

A partial explanation of the relative superiority of the low-dispersion instrument is that it may be due to the fact that the three-prism plates were mostly made on different dates and at varying hour angles, though never far from the meridian; while the one-prism plates were made in four groups only, the plates being obtained consecutively and probably under similar conditions in each of the groups.

In any case it seems to be evident that radial velocity determinations of second-type stars may be made with low-dispersion instruments with an accuracy not much less than that obtainable with the high-dispersion instruments at present in use. This fact, if considered established by the present investigation, is one of much importance, as it admits the carrying of the spectrographic survey of the heavens to stars more than a magnitude fainter than those at present available.

It may be of interest to compare the values of the probable errors of a region and plate in the solar-type star *Arcturus* with those obtained in my investigations on the effect of slit-width,[2] of the probable errors of an average line and of a single plate in the case of the early-type star *β Orionis*. I will in this case take the mean of the values obtained for the three slit-widths 0.025, 0.038, and

[1] *Journal R.A.S. Canada*, **3**, 287, 1909.

[2] *Astrophysical Journal*, **28**, 259, 1908; *Trans. Royal Society of Canada*, **3**, 209, 1909.

0.051 mm, thus using 18 plates with III L, 30 plates with III R, and 30 plates with I.

PROBABLE ERRORS

Spectrograph	Line of Average Weight	Single Plate
III L. 10.1	±2.20	±1.17
III R. 20.2	±2.47	±1.53
I 33.5	±3.18	±2.16

These results are relatively to one another in substantial agreement with those obtained in the present investigation though not quite so favorable to the low dispersion. There seems hence to be no reasonable doubt that the accuracy of radial velocity determinations does not by any means proportionately diminish with decrease of dispersion.

It is evident therefore that the high probable errors of single observations obtained in our work on spectroscopic binary orbits must be due, not to the small dispersion employed giving results relatively less accurate than those obtained with high-dispersion instruments, but to the character of the lines in the spectra, with the resultant high errors of measurement. In many cases also they are probably due to abnormal conditions in the orbit causing deviations from velocity-curves due to simple elliptic motion, thus giving higher residuals.

I have tabulated below the probable error of an average observation of the velocity of the brighter stars of some of the spectroscopic binary systems as determined here and at the Allegheny and Lick observatories and it will be noticed at once that the accuracy very rapidly diminishes as the spectrum lines become broader and with orbits containing abnormal secondary or other effects. The slightly lower values obtained at Allegheny for the probable errors are likely due to the fact that many of their spectra were made on the fine-grained Seed 23 instead of the coarser-grained Seed 27 plate used at Ottawa.

None of the orbits determined with low dispersion at Ottawa and Allegheny is of stars with solar-type spectra, so that no measure can be thus obtained of the relative accuracy of this

dispersion. Three solar-type binaries observed at the Lick Observatory with a single-prism spectrograph show fairly accurate results, especially *W Sagittarii*, with a plate error of ±0.90 km, and

PROBABLE ERRORS OF SINGLE OBSERVATIONS

Star	Spectral Type	Probable Error Average Plate	Dispersion	Observatory	Remarks
7 Camelopardalis	A 2 F	±2.4	One-prism	Ottawa	Lines good
α Draconis	VII a	3.4	One-prism	Ottawa	Lines good
θ Aquilae	VII a	4.0	One-prism	Ottawa	Changing elements?
B. D.−1°1004	B 3 A	5.2	One-prism	Ottawa	Broad line
α Coronae	VIII a b	5.4	One-prism	Ottawa	Lines broad and asymmetric
φ Persei	L	4.1	One-prism	Ottawa	Abnormal secondary effect
ε Herculis	VII a	6.4	One-prism	Ottawa	Abnormal secondary effect
ι Orionis	B, I b	6.8	One-prism	Ottawa	Lines diffuse and asymmetric
ψ Orionis	B, I b	6.8	One-prism	Ottawa	Lines very broad
τ Tauri	B, I b	10.8	One-prism	Ottawa	Lines very broad
β Orionis	VI c	2.0	Three-prism	Ottawa	Lines good
		3.2	One-prism	Ottawa	Lines good
η Virginis	VIII a	2.4	Three-prism	Ottawa	Lines fair
η Boötis	XIV a	0.50	Three-prism	Ottawa	Numerous well defined lines
π⁴ Orionis	IV a	1.7	One-prism	Allegheny	Lines very good
ζ¹ Lyrae	A, I a 2	2.0	One-prism	Allegheny	Lines well defined
α Andromedae	VIII P	2.5	One-prism	Allegheny	Lines good
β Aurigae	VIII a	3.2	One-prism	Allegheny	Lines sharp and narrow
θ Aquilae	VII a	3.4	One-prism	Allegheny	Lines fair
δ Librae	VII a	4.7	One-prism	Allegheny	Ill-defined lines
α Coronae	VIII a b	4.9	One-prism	Allegheny	Broad lines
2 Lacertae	A, I b	5.3	One-prism	Allegheny	Fair lines
u Herculis	A, I b	8.0	One-prism	Allegheny	Broad lines
α Virginis	III b	9.9	One-prism	Allegheny	Very broad and diffuse
η Pegasi	XIV a	0.47	Three-prism	Lick	Good lines
α Aurigae	XIV a	0.50	Three-prism	Lick	Good lines
λ Andromedae	XV a	0.50	Three-prism	Lick	Good lines
β Herculis	XV a	0.52	Three-prism	Lick	Good lines
ε Pegasi	XII a	0.56	Three-prism	Lick	Good lines
α_2 Geminorum	VIII a	0.64	Three-prism	Lick	Good lines
α_1 Geminorum	VIII a	0.79	Three-prism	Lick	Good lines
ω Draconis	F 5 G	0.75	Three-prism	Lick	Diffuse lines
θ Draconis	XIII a	0.87	Three-prism	Lick	Diffuse lines
RT Aurigae	G	0.82	Three-prism	Lick	Diffuse lines
η Aquilae	XIV a c	0.85	Three-prism	Lick	Diffuse lines
W Sagittarii	F 5 G	0.90	One-prism	Lick	Fair lines
X Sagittarii	F 8 G	1.80	One-prism	Lick	Fair lines
Y Sagittarii	G	2.10	One-prism	Lick	Fair lines

which if three or four discrepant observations are omitted reduces to ±0.55 km. This is of the same order as the probable error of a single-prism plate of *Arcturus* as determined here (±0.70 km).

The probable errors of single observations of binary and constant-velocity solar-type stars with three-prism dispersion both at Lick and Ottawa seems to be very close to half a kilometer and in cases where it is greater it is apparently due to the poorer quality of the spectra for measurement. Three cases in which the probable error of a plate is less than half a kilometer are known to me. At Ottawa a series of 11 plates of *β Geminorum* gave a probable error of ±0.40 km per second. When observations are limited to certain hour angles and special precautions taken, as at Bonn by Küstner in determination of the solar parallax, 16 plates of *Arcturus* gave a probable error of a single plate of ±0.22 km; and at the Royal Observatory Cape of Good Hope, 22 plates of *β Geminorum* gave a probable error of ±0.34 and 55 plates of *α Boötis* of ±0.42 km. There is no doubt, though no values have been published, that the work at the Lick and Yerkes Observatories on solar-type constant-velocity stars is equally accurate.

It seems to me therefore that we may safely draw the following conclusions from the preceding discussion:

I. The accuracy of determination of the radial velocity of stars of solar type by means of spectrographs of different dispersions is not, as would be expected, inversely proportional to the dispersion, but in the cases under discussion only a small increase of probable error, 40 per cent, takes place when the dispersion is divided by three. As the relative exposures required are as about five to one, it is evident that stars more than a magnitude and a half fainter become available.

II. The probable error very rapidly increases with the increase in diffuseness of the lines in early-type stars, varying in low-dispersion spectrographs from about ±2 to ±11 km per second. Experience in work with these stars has convinced me that the whole of this error is not due to the accidental error of pointing, but that in many cases some physical cause in the star's atmosphere is responsible for a considerable part of the discrepancy.

III. The results of this and other investigations show that the

probable error of a single ordinary observation of a good second-type star with the usual three-prism dispersion is in the neighborhood of 0.5 km per second. When greater than this, it indicates a spectrum with poorer lines, and when less, that special precautions and limitations were adopted in the making and measurement of the spectra. It is also shown that the probable error of determination with a thoroughly stable one-prism instrument of one-third the dispersion is for solar-type spectra measured on the spectrocomparator about ± 0.70 km per second, and for spectra of earlier type varies from about ± 2 to ± 11 km per second.

IV. Generally speaking the major part of the errors in solar-type stars are due to systematic displacements of the lines as a whole owing to flexure, temperature changes, imperfect adjustments of the optical parts, faulty guiding, or other causes, and the accidental errors of pointing are responsible generally for only one-third or less of the total error. In the case of early-type stars the systematic displacements due to instrumental conditions will probably be approximately the same while the errors of pointing are correspondingly increased.

The interest of the Director, Dr. W. F. King, in this work is hereby gratefully acknowledged.

DOMINION OBSERVATORY
OTTAWA, CANADA
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THE COLLIMATION OF THE CORRECTING LENS

By J. S. PLASKETT

When the new single-prism spectrograph was brought into use it was noticed that the star spectra obtained were unusually weak in the violet. Although the absorption of the new prism of 51 mm aperture is somewhat greater than that of the one previously in use (35 mm aperture), this difference is by no means sufficient to account for the difference in intensity at the violet in the two cases.

In many star spectra the lines to the violet of $H\delta$ are important and the best measures obtained are frequently those of the K line. Moreover, greater accuracy should be obtained in this part of the spectrum on account of the greater linear scale.* It was hence important to discover, if possible, the cause of and the remedy for this difficulty.

It was necessary, when the new spectrograph was attached to the telescope, to make some changes in the attachment of the correcting lens, and there was a possibility that the distance of the lens from the focus had become changed and the form of the color-curve altered sufficiently to throw the image in violet light considerably beyond the slit. However, a redetermination of the color-curve by Hartmann's method of extra-focal exposures showed that the minimum focus was at about λ 4325, that the focal points for light at λ 3930 and λ 4700 were each about three millimeters beyond that at λ 4325, and moreover that the position of the slit was the most favorable for obtaining uniform intensity of the photographed spectrum from $H\beta$ to K.

These two possible causes having thus been disposed of, the next tested was the guiding apparatus. The visible image produced by the combination of visual objective and correcting lens consists of a more or less condensed nucleus of blue-green light surrounded by an extended penumbra or halo of reddish light, and one guides by the blue-green nucleus while the slit is rendered partly visible by the penumbra. It is not possible, owing probably to chromatic aberration, to get both sharp at the same time, and there is usually

considerable parallax, due partly to this cause and partly to the fact that the visual blue-green image is really a millimeter or more beyond the slit. It is possible, therefore, although the image may be kept apparently central, that its effective part is really to one side or other of the slit, producing diminished intensity of spectrum. However, a careful readjustment of the reflecting prism and guiding apparatus produced no perceptible improvement.

The following test was then made to determine whether guiding with the image apparently above or below the slit would have any effect. After the governors had been so adjusted that the star image drifted the width of the star window along the slit in 20 seconds, four spectra of the bright star *α Aquilae* were made, with an exposure on each of 60 seconds, 3 times drifting, side by side on the one plate. In the first of these the star image was kept bisected by the upper edge of the slit opening (as seen in the guiding telescope), in the second the image was kept central, in the third bisected by the lower edge, and in the fourth kept entirely below and just touching the lower edge of the slit. The result of this test is shown in the figure at *A* where *a*, *b*, *c*, *d* represent the four positions in guiding. It will be noticed at once that the point of maximum intensity moves toward the violet as the image is moved down. A little consideration renders it evident that this effect is due to a sort of dispersion of the image, that the violet part of it falls above the blue-green by more than the width of the slit. This cannot be detected by the eye on account of the very low visual intensity of the violet light. As a matter of fact the guiding must be done almost entirely by the image formed by light of wavelengths between λ 4600 and λ 4900. To the violet of this region it is too faint, and to the red the image is too far out of focus for any effect on the eye. It is evident from the figure that it is only when the image formed by this light seems to be entirely below the slit that the maximum intensity in the violet is obtained, and that in consequence the violet image is on the slit.

Such dispersion of the image may be assigned to one or more of three causes: (*a*) to faulty squaring of the objective; (*b*) to atmospheric dispersion; (*c*) to imperfect collimation of the correcting lens.

A test of the objective showed it to be in correct adjustment, and moreover throwing it considerably out of square had no appreciable effect on the appearance of the image, or on spectra made under the same conditions as before.

That it was not wholly due to atmospheric dispersion was at once proved by the same test, as described above on *α Aquilae*, applied to a star near the zenith, where a similar effect was produced.

That incorrect collimation of the corrector was the principal cause was finally shown when, after a change in collimation, the test gave a different arrangement of the intensity in the four spectra. The correcting lens, which was specially designed,[1] has an aperture of 102 mm (4 inches) and is placed 150 cm (59 inches) within the focus. It is mounted in a tube whose lower end is held in a flange which screws into the telescope adapter, while near the upper end a guide ring holds it central. It was carefully collimated with both the old and the new spectroscope by pointing the telescope to the zenith, removing the objective, and hanging a steel piano wire centrally through the objective cell, the correcting lens cell, the slit, and the collimator lens cell. It is possible that after collimation there may have been some movement of the spectroscope before it was finally rigidly fastened into position, and that this was sufficient to produce the observed effect, for, as will be seen later, a displacement of a millimeter is sufficient to produce marked changes in the distribution of intensity in the spectrum.

Some further tests showed the necessity for having the collimation of the correcting lens made adjustable. Consequently the upper end of the tube was made movable transversely to the slit by applying a screw of $\frac{1}{16}$ inch ($1\frac{1}{2}$ mm) pitch, with a divided head, to the guiding ring above mentioned. No longitudinal adjustment was provided, as any dispersion along the slit will not tend to throw any part of the image to one side or other of the slit opening.

A series of tests, similar to the one described above, was then made at different settings of the adjusting screw of the corrector tube, and it was soon discovered that the correcting lens was very

[1] *Astrophysical Journal*, **28**, 139, 1908; *Report of Chief Astronomer*, 1908, p. 73.

much more sensitive to changes in collimation than had been imagined. Plate XIV shows tests at three settings, *B*, 1.6; *C*, 2.0; *D*, 2.4, of the adjusting screw, the distance between successive positions being 0.6 mm. The spectra are of the star *α Aquilae*, and the exposures, which are of 60 seconds each, are made with the star near the meridian and the telescope west of the pier. The slit was 0.051 mm wide and the image was guided so that *a*, *b*, *c*, *d* refer to the same positions as at *A*.

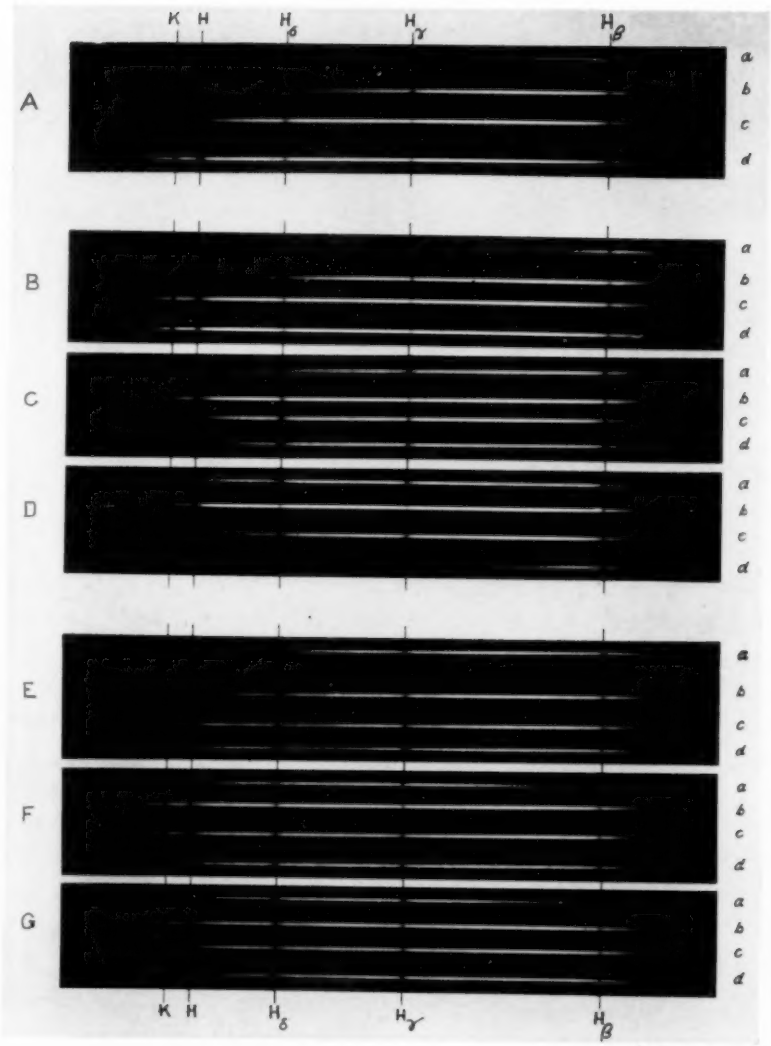
The criterion for determining the best position of the correcting lens must evidently be the intensity toward the violet end of the spectrum, for, owing to the greater photographic effect in the blue-green and the expanded image there due to the form of the color-curve of the corrector, a slightly asymmetric position of the image on the slit will have little effect on the intensity. It will be noticed then that *c* in *B*, *b* in *C*, and *a* in *D* are the spectra which have the greatest extension and intensity in the violet and are evidently therefore the positions where the violet part of the image was central on the slit. Hence in position *B*, setting 1.6, the violet image was apparently above; in position *D*, setting 2.4, apparently below the blue-green image on which guiding is necessarily performed; while in *C*, setting 2.0, they are superposed, as the greatest intensity in the violet is given when the image is kept central.

C, at setting 2.0 of the corrector screw, is therefore the position, not necessarily (as the star is not at the zenith) of exact collimation of the correcting lens, but the position where the dispersion produced by the atmosphere is just counteracted by the opposite dispersion caused by a slight departure of the corrector from exact collimation.

If we now look at the lower part of the figure, spectra made in a similar way of the same star near the meridian, but with the telescope east of the pier, where *E* is at setting 0.8, *F* 1.2, and *G* 1.6, we find that the best position *F* is at a setting of the correcting lens 0.8 rev., or 1.25 mm, distant from the best position west of the pier.

As the atmospheric dispersion acts in the same direction in each case, the change in collimation must be due to flexure of the tube

PLATE XIV



TESTS OF COLLIMATION OF CORRECTING LENS

of the telescope, which is sufficient to displace the correcting lens at the declination +8°.6 by 0.6 mm ($\frac{1}{40}$ of an inch), from the line joining the center of the objective and the slit. This is not unreasonably large when we consider that the correcting lens is 150 cm from the slit. A further test on *α Cygni*, which at the meridian is only half a degree from the zenith, showed the best position of the corrector to be at setting 1.6 midway between the others.

Consequently, assuming this change of collimation to be due to flexure, and that approximately this is proportional to the sine of the zenith distance, a table of the corrector settings for every ten degrees of zenith distance, from 0° to 70°, was computed and will be used, after some further tests at intermediate points and at large hour angles, for every spectrum taken.

The necessity of this is very clearly shown by the marked dispersion of the image indicated in *B*, *C*, *D* and *E*, *F*, *G* for a difference of position of the correcting lens of 0.6 mm. Indeed further tests showed that a change of collimation of only 0.25 mm can easily be recognized by the distribution of intensity in the spectra, and it is evident that, for the best results, we must insure very exact collimation. These tests show that a photographic correcting lens for visual objectives has no field, that it must be used along the axis. If the cone of light from the objective, which is 99 mm in diameter, falls even 0.25 mm to one side of the center of the correcting lens, the star image is dispersed in such a way that the image in violet light falls to the same side of the blue-green image. Estimating from the tests made, the dispersion or separation of the blue-green and violet images is approximately 0.05 mm for each 0.75 mm the corrector is moved.

These experiments have seemed worth recording, as showing how very carefully the corrector must be collimated, even to such an extent that the departure from collimation due to flexure of the telescope tube must be accurately compensated. It is convincingly demonstrated by the figure that a departure from collimation of only 0.6 mm (0.025 inch) will much increase, nearly double, the exposure time required for the violet end of the spectrum. Moreover, there is a further even more important consideration, that systematic displacement of the lines is less likely to occur

under accurate collimation. Uniform illumination of the collimator lens by light of every wave-length (necessary to prevent possible systematic displacements) can be produced only when the images in light of all the wave-lengths used fall centrally on the slit.

It gives me much pleasure to express my appreciation of the interest shown by the Director, Dr. King, in this work.

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ON THE REVERSAL OF SPECTRAL LINES[1]

By H. KONEN

1. In a repetition of Pflüger's[2] experiment on the absorption of luminous hydrogen, A. Ladenburg[3] observed certain phenomena of reversal when he used high dispersion, which he regarded as a proof that the quotient E/A, emission over absorption, is not constant within the line investigated. He further concluded that the radiation of hydrogen is not a temperature radiation.

In contrast, Pflüger[4] has shown that the explanation offered by Ladenburg is a possible one, but not a necessary one, and that the observed phenomena might equally well be explained for a constant E/A, if we take into account the fact that Ladenburg's emitting tube furnishes a spectrum by no means equivalent to the continuous spectrum of a black body, but that it has diffuse maxima at the positions of the hydrogen lines. He accordingly identified the phenomenon with the multiple reversal of spectral lines often observed and discussed by spectroscopic observers, and at the same time he has contributed toward an explanation of the phenomenon.

On first looking at Ladenburg's illustrations, I was reminded of the phenomena of reversal previously described by Konen and Hagenbach,[5] and I immediately communicated to Professor Kayser of Bonn a computation in complete agreement with the results of Pflüger. The latter then sent me his manuscript, and I accordingly laid aside my notes. It now appears to me, however, that they may have some interest in connection with those of Pflüger, inasmuch as they propose an entirely different way, and also permit a somewhat simpler and more rigorous manner of

[1] Translated from advanced proof sheets of an article appearing in the *Physikalische Zeitschrift*, **11**, 663, 1910.

[2] *Annalen der Physik* (4), **24**, 515, 1907.

[3] *Verhandlungen der deutschen physikalischen Gesellschaft*, **10**, 550, 1908; **12**, 54, 1910.

[4] *Ibid.*, **12**, 208, 1910.

[5] *Astrophysical Journal*, **19**, 110, 1904.

regarding the reversal phenomena of spectral lines than do the methods of representation hitherto given.

2. Let us designate by E_2 the emission of the source of light which is to serve as an absorbing medium, that of the emitting source by E_1, while A_2 and A_1 may signify the corresponding absorptions, and e the emissive power of the absolutely black body. The source of light I is then observed through the luminous gas II. Let it further be assumed that Kirchhoff's law holds good within the spectrum, i.e., $E_2/A_2=e''$, $E_1/A_1=e'$, in each instance for a definite temperature and wave-length. The resulting emission E then obviously is the value

$$E_2+E_1(1-A_2)=E\,,$$

and it may be represented by the curve $E=f(\lambda)$. A reversal will occur at each point where this curve has a minimum. Whether this reversal will be perceived in practice as such, i.e., as a relatively dark band on a brighter background or not, will depend upon the method of observation and on the relative intensity of this minimum. A decision on this point is, however, a secondary matter to be settled from case to case.

In the experiments on reversal which we are now considering, we are dealing throughout with spectral lines, that is, we may assume that the function E_2 has pronounced maxima within which the course of E_2 is an "ordinary" function.[1] The same thing is true of E_1. So we are able to follow the course of E with the aid of $dE/d\lambda$. It is evident, in order to have a reversal occur, that $dE/d\lambda$ must be at least once equal to zero in the region of an emission line of body II. Therefore we must have

$$\delta E/\delta\lambda=\delta E_2/\delta\lambda+\delta E_1/\delta\lambda-\frac{\delta}{\delta\lambda}\frac{(E_1E_2)}{e''}=0\,.$$

Within the narrow maxima corresponding to a line, e'' may be regarded as constant. The first condition therefore reads

$$\frac{\delta E_2}{\delta\lambda}\left(1-\frac{E_1}{e''}\right)+\frac{\delta E_1}{\delta\lambda}\left(1-\frac{E_2}{e''}\right)=0\,. \qquad (1)$$

[1] The spectrum must be reproduced in order that E_2 may be investigated. This alone furnishes a justification for the assumption made.

3. It is not permissible that we should now regard every minimum in the course of E as a reversal. Minima and maxima in the course of E_1 and E_2 are to be excluded. It will therefore henceforth be assumed that neither E_1 nor E_2 possesses more than one maximum, and that they have no minimum. In other words, that the lines under consideration are simple and not compound. Further, let E_2 be a line and not a continuous spectrum. Finally, for simplicity, let us assume that the centers of the emission lines of E_1 and E_2 coincide. It is obvious that this involves no serious limitation. We may now state that $e''-E_2$ is always positive, never zero, and that the differential quotients $\delta E_1/\delta\lambda$ and $\delta E_2/\delta\lambda$ have the same sign. It therefore follows that the first member in equation (1) must have the opposite sign from the second for all points in which $\delta E_1/\delta\lambda$ and $\delta E_2/\delta\lambda$ do not disappear. This case always occurs at the center of the lines. We shall return to it soon.

If both differential quotients differ from zero, we shall obtain as the first condition for the occurrence of a reversal the well-known relation $E_1>E_2$, i.e., light-source I must have a higher temperature than source II.

4. The case remains where one or both of the differential quotients vanish. We have the relation $\delta E_1/\delta\lambda=0$, either when E_1 is a constant, therefore if I emits a continuous spectrum, or when E_1 has a maximum or a point of inflection. If I emits a continuous spectrum, then we have

$$\frac{\delta E_2}{\delta\lambda}(e''-E_1)=0. \qquad (2)$$

This equation is always fulfilled if $\delta E_2/\delta\lambda=0$, or $e''-E_1=0$, or therefore at the center of the line of gas II, or if both bodies have the same temperature. If a reversal now occurs, $\delta E/\delta\lambda$ must be negative below the center and positive above the center. We therefore have for $\lambda<\lambda_0$, $\frac{\delta E_2}{\delta\lambda}(e''-E_1)<0$, for any selected points; therefore, since $\delta E_2/\delta\lambda$ is positive, $e''-E_1$ must be less than zero, therefore $E_1>e''$, and accordingly the temperature of I must be higher than that of body II. At the same time it results that equation (2) can be fulfilled only for the position $\lambda=\lambda_0$, since E_1

is assumed as constant. The result, therefore, is that against a continuous spectrum only a simple reversal can take place which has its maximum at the place of the maximum of emission, and that it occurs always and only when $E_1 > e''$. Therefore, if we observe a multiple reversal against a continuous background, we are to assume that E_2/A_2 varies, and that E_2 does not follow Kirchhoff's law, it being assumed that we are dealing with a simple line.

5. E_1 can have a maximum only when it consists of a single line. In the middle of this line $\delta E_1/\delta\lambda = 0$, so that equation (1) is satisfied, being identical. In order that a reversal should be visible at the center of E, the differential quotient $\delta E/\delta\lambda$ must be negative on one side of the center below λ_0, and positive above the center, within a finite range. We therefore obtain from equation (1)

$$\Delta = \frac{\delta E_2}{\delta\lambda}(e''-E_1)+\frac{\delta E_1}{\delta\lambda}(e''-E_2)<0\,. \qquad (3)$$

The second number is now positive and similarly $\delta E_2/\delta\lambda$, whence $e''-E_1$ must be less than zero. We therefore again obtain as a necessary, but this time not sufficient condition, that $E_1 > e''$, i.e., the temperature of I must be higher than that of II in order that a reversal at the center can be possible.

If E_1 has a point of inflection, but E_2 has not, then $\delta E_1/\delta\lambda = 0$, whence according to equation (1) $e''-E_1 = 0$, inasmuch as $\delta E_2/\delta\lambda$ does not vanish. Below this point $E_1 < e''$, and above it $E_2 > e''$, in accordance with our assumption as to the lines in question. If a minimum or a reversal is to occur, then must $\delta E/\delta\lambda < 0$, hence

$$\Delta = \frac{\delta E_2}{\delta\lambda}(e''-E_1)+\frac{\delta E_1}{\delta\lambda}(e''-E_2)<0\,.$$

All the terms are here positive, and the equation is impossible. Points of inflection of E_1 therefore furnish no reversal. The same is true if E_2 has a point of inflection while E_1 has not. We therefore obtain the condition

$$\frac{\delta E_1}{\delta\lambda}(e''-E_2)=0\,,$$

which is impossible of fulfilment.

The last case to be considered would be that where both E_1 and E_2 have one point of inflection. Condition I is then fulfilled. Below this point, we must in case of a minimum have $\Delta<0$, and above it $\Delta>0$. Since all the members of Δ are positive, with the exception of $e''-E_1$, this must be negative, and we therefore again obtain as a necessary but insufficient condition for the occurrence of a reversal, $E_1>e''$, i.e., the temperature of E_1 must be higher than that of E_2.

6. The question arises what are the further conditions required to produce a reversal in the other cases. Inasmuch as $E_2<e''<E_1$, equation (1) may be written

$$+\frac{\delta F_2}{\delta\lambda}F_1+\frac{\delta F_1}{\delta\lambda}F_2=\frac{\delta}{\delta\lambda}(F_1F_2)=0$$

if we place

$$E_1-e''=F_1,\ e''-E_2=F_2\ .$$

A minimum therefore occurs if the product $F_1\times F_2$ possesses a mimimun. F_1 is here a function which considerably increases from the edge of the line to its center, while F_2 decreases. If F_1 and F_2 are given, as in the experiment, the positions of reversal can therefore be reckoned when e'' is known. But here the question arises whether such cases are really thinkable. It is, however, the fact that quantities F_1 and F_2 can always be found, one of which continuously increases while the other continuously decreases, so that their product has several maxima and minima. We can convince ourselves of this by a drawing or by forming the quantities

$$\frac{\delta(F_1F_2)}{\delta\lambda}$$

and

$$\frac{\delta^2(F_1F_2)}{\delta\lambda}\ .$$

We therefore obtain

$$\frac{\delta F_1}{\delta\lambda}:\frac{\delta F_2}{\delta\lambda}=-F_1:F_2$$

and by appropriate disposition of the second differential quotients

$$\frac{\delta^2(F_1F_2)}{\delta\lambda^2}$$

make

$$\frac{\delta^2(F_1F_2)}{\delta\lambda^2}$$

positive as well as negative for given positions. Hence, it follows that multiple reversals are always possible, even when Kirchhoff's law is valid, i.e., always when the product F_1F_2 has several maxima.[1]

7. We may now also determine in which case a reversal occurs in the center of the two lines, or in two points of inflection corresponding to the same value. Inasmuch as

$$\frac{\delta F_1}{\delta\lambda}=\frac{\delta E_1}{\delta\lambda}=0\,,\quad \frac{\delta F_2}{\delta\lambda}=-\frac{\delta E_2}{\delta\lambda}=0\,,$$

therefore

$$\frac{\delta^2(F_1F_2)}{\delta\lambda^2}$$

takes the value

$$F_1\frac{\delta^2F_2}{\delta\lambda^2}+F_2\frac{\delta^2F_2}{\delta\lambda^2}>0\,.$$

Now F_1 evidently has a maximum at the center of the line, whence

$$\frac{\delta^2F_1}{\delta\lambda^2}<0\,.$$

Conversely, F_2 has a minimum at the same place. We therefore get a dark center, without regard to the sign, of

$$F_1\frac{\delta^2F_2}{\delta\lambda^2}>F_2\frac{\delta^2F_2}{\delta\lambda^2}\,,$$

and a bright center, if the reverse holds true. This condition is obviously identical with equation (3). If E_1 and E_2, and hence also F_1 and F_2 have points of inflection, then

$$\frac{\delta^2F_1}{\delta\lambda^2}\,,\quad\frac{\delta^2F_2}{\delta\lambda^2}$$

also vanish, i.e., F_1F_2 also has a point of inflection. It therefore follows that no reversal can occur at a point where E_1 and E_2 have points of inflection.

8. The results of the whole discussion may be summarized in the following theorem.

[1] I am indebted to the kindness of Professor Dehn of Münster for examples of what has been said, e.g.: $F_2=1/\lambda$, $F_1=y$, where y is defined by the equation $\frac{(y-\lambda)}{\sqrt{2}}=\frac{1}{\sqrt{2}}\sin\rho\left(\frac{y+\lambda}{2}\right)=0$. By an appropriate choice of ρ we may obtain as many maxima and minima as may be desired in a given interval of λ.

In order that reversals should occur, it is necessary that the emitting source of light should have a higher temperature than the absorbing source of light.

If the emitting body furnishes a continuous spectrum, then only a simple reversal is possible, and it occurs if $E_1 > e''$.

If the emitting body gives a spectrum with a pronounced maximum near the line of the absorbing body, a reversal will occur at the center of the line if $E_1 > e''$, and, disregarding signs, if

$$F_1 \frac{\delta^2 F_2}{\delta \lambda^2} > F_2 \frac{\delta^2 F_1}{\delta \lambda^2} .$$

Since $\delta F_2/\delta\lambda$ and $\delta F_1/\delta\lambda$ are equal to zero, this may also be written, with disregard for the sign,

$$|F_1 \rho_1| > |F_2 \rho_2|, \tag{4}$$

where ρ designates the radius of curvature at the center. This means that the occurrence of a reversal is favored the greater the difference $E_1 - e''$, and the flatter the curve E_1; or the sharper the emission line and the smaller the difference $e'' - E_2$; that is to say, the more the maximum of E_2 approaches the emission of the black body.

Away from the center of the line reversals are not possible at places where one or both of the curves have points of inflection. They may, however, occur at all places where the product $F_1 \cdot F_2$ has a minimum. Without respect to forms of E_1 and E_2, which are physically possible, this may be designated as always possible. An n-fold reversal (n black lines) will therefore be observed if

$$\frac{\delta(F_1 F_2)}{\delta\lambda} \quad \ldots .$$

disappears $(2n-1)$ times, and if n minima and $n-1$ maxima occur.

9. It is of interest to examine from the point of view of the above rules the forms of the energy-curves of simple lines derived theoretically. One example will suffice. We choose the function e^{-x^2} as used by Lord Rayleigh[1], which may for our purpose be written

$$E_1 = a_1 e^{-k_1^2(\lambda - \lambda_0)^2}$$

[1] Kayser's *Handbuch der Spectroscopie*, Bd. **2**, 343.

when λ_0 denotes the center of the line. We now have

$$F_1F_2=(E_1-e'')(e''-E_2)=\left(a_1e^{-k_1^2(\lambda-\lambda_0)^2}-e''\right)\left(e''-a_2e^{-k_2^2(\lambda-\lambda_0)^2}\right).$$

Therefore, in order that a reversal should be possible for the center of the line, $E_1>e''$, hence $a_1>e''$, and also $a_2<e''$. Further, inasmuch as E_1 corresponds to the higher temperature, its line must be the broader, hence $k_1^2<k_2^2$. It is evident that at the center of the two lines the condition

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0$$

is fulfilled. The condition in equation (4) gives

$$|F_1\rho_1|>|F_2\rho_2|,$$

that is, since for $\lambda=\lambda_0$

$$\frac{\delta^2F_1}{\delta\lambda^2}=-2k_1^2a_1,$$

$$\frac{k_2^2a_2}{k_1^2a_1}>\frac{e''-a_2}{a_1-e''}.$$

In consequence of the condition

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0,$$

and since neither $\delta E_1/\delta\lambda$ nor $\delta E_2/\delta\lambda$ can vanish, we have away from the center of the line

$$\frac{k_1^2E_1}{k_2^2E_2}=\frac{F_1}{F_2}.$$

This condition can evidently always be satisfied if

$$e''=\frac{(k_1^2+k_2^2)E_1E_2}{k_1^2E_1+k_2^2E_2}.$$

If we write this equation in the form

$$\frac{k_1^2}{E_2}+\frac{k_2^2}{E_1}=\frac{k_1^2+k_2^2}{e''},$$

and recall that both E_1 and E_2 increase from the edge toward the center of the line, and further that e'' is fixed, then we see that condition (1) can at most be fulfilled only once on each side of the line. In order to decide whether a minimum, that is, a reversal, can occur, we form the second derivative of F_1F_2. Regarding the relation

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0,$$

this takes the value

$$4(\lambda-\lambda_0)^2[e''(k_1^4E_1-k_2^4E_2)-(k_1^2+k_2^2)E_1E_2]>0,$$

or, with the omission of the positive factor $4(\lambda-\lambda_0)^2$,

$$k_1^4E_1+k_1^4E_2>(k_1^2+k_2^2)^2E_1E_2\frac{1}{e''}.$$

If we introduce the value of e'' found above, we get

$$k_1^4E_1+k_2^4E_2>(k_1^2+k_2^2)(k_1^2E_1+k_2^2E_2)$$

or

$$0>+(E_1+E_2)k_1k_2,$$

which is not possible, since the right-hand member contains only positive quantities which cannot vanish.

This accordingly shows that two lines of the form e^{-x^2} cannot furnish a double reversal. Inasmuch as we may assume that the majority of the simple spectral lines have an approximately similar distribution of intensity, this explains at the same time why genuine double reversals are relatively so infrequent.

10. We may convince ourselves in a geometrical manner that the occurrence of double reversals is favored by an intensity-curve for the two lines in which $\delta E_1/\delta\lambda$ and $\delta E_2/\delta\lambda$ are subject to decided variations. Such cases will be observed particularly when non-homogeneous layers of different temperatures are located one behind another, as in an arc-lamp burning irregularly, or in a Geissler tube having different diameters, the discharge being stratified.

Finally, we may readily see what effect is produced if the intensity of E_1 is diminished to the p^{th} part, for instance by inserting a Nicol prism. If a reversal occurs without the Nicol prism at the position $\lambda=\lambda_1$, then at that point

$$\frac{\delta(F_1F_2)}{\delta\lambda}=0.$$

If E_1 is diminished to its p^{th} part, this furnishes the condition $E_1>pe''$. Consequently the new value of E_1 will lie nearer to the center of the line if the reversal is to persist. The reversal therefore moves with the position of the Nicol from the edge of the line toward the center.

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Minor Contributions and Notes

THE INTERNATIONAL UNION FOR CO-OPERATION IN SOLAR RESEARCH

The fourth meeting of the Union was held in four formal sessions on the three days August 31, September 1, and September 2, 1910, on Mount Wilson, California, under the auspices of the Solar Observatory of the Carnegie Institution. The general arrangements for the sessions were made by the Executive Committee of the Union, Messrs. Arthur Schuster, George E. Hale, and A. Riccò.

Most of those attending the meeting arrived in Pasadena on August 28. The Pasadena offices, laboratory, and shops of the Mt. Wilson Solar Observatory were inspected on the morning of August 29. On that afternoon the members and guests were entertained at a garden party by Mr. and Mrs. Hale. August 30 was devoted to the ascent of Mount Wilson, and to the inspection of the buildings and instruments of the observatory. At the regular sessions on the three following days no formal papers were presented, the time being fully occupied by committee reports and their discussion. An informal address was given by Mr. Hale at the opening of the session, in which, in addition to words of welcome, he explained some of the methods and results of the spectrographic, spectroheliographic, and magnetic study of the sun-spots at Mount Wilson. On the evening of August 31 an informal lecture on the solar constant and the recent methods for its determination was given by Mr. Charles G. Abbot of the Smithsonian Astrophysical Observatory. Mr. J. C. Kapteyn of Groningen, research associate of the Carnegie Institution, gave on the evening of September 1 an informal lecture on star streams among the stars of the *Orion* type.

Ample opportunity was given during the three days and four nights spent on the mountain for the visitors to examine the instruments of the observatory while in operation, and particularly to view celestial objects with the five-foot reflector, with the assist-

ance of its constructor, Mr. G. W. Ritchey. The station of the Smithsonian Observatory, maintained on Mount Wilson during the summer season for spectrobolometric and other observations of the solar radiation, was also open for inspection daily.

The members of the Union and guests were entertained at a final banquet given by Mr. and Mrs. Hale on the evening of September 3 at Hotel Maryland, Pasadena.

Some of the more important resolutions adopted at the meeting are given herewith, but this is not to be regarded as in any sense an authoritative report of the meeting. The official record of the conference will be given in full in the next volume of the *Transactions* of the Union.

I. *Resolutions adopted as reported by the committee on standards of wave-length:*

1. In the region of the spectrum in which three independent measurements, by the interferometer method, of the lines of the iron arc are available, i.e., between λ 4282 and λ 6494, the arithmetical mean of the three measurements shall be adopted as definite international standards of the second order, provided that there is sufficient agreement between them.

2. The committee is given authority to publish these standards as soon as possible.[1]

3. For the part of the spectrum in the neighborhood of λ 5800, where the number and character of the iron lines is not satisfactory, the committee proposes the use of barium lines as additional standards.

4. The laboratories or observatories possessing first-rate concave gratings are invited to determine by interpolation, as soon as possible, standards of the third order in the spectrum of the iron arc, within the above range of the spectrum, λ 4282 to λ 6494.

5. The measurement of standards of the second order shall be extended to shorter and longer wave-lengths, and the arithmetical mean of three independent determinations shall be adopted as secondary standards.

6. Standards of the third order shall then be obtained in the manner indicated.

[1] See p. 215 of this number of this *Journal*.

7. The above system of standards shall be called the international system, the unit on which it is based being called the international unit (I.A.) as defined by the Conference of 1907.

8. It is very desirable that in different laboratories possessing concave gratings of the first quality photographs of the arc, spark, and solar spectrum, and new measurements according to the international system, shall be made as soon as possible.

II. *Committee on the measurement of solar radiation:*

The report of the committee was drafted and presented by Charles G. Abbot of the Smithsonian Observatory at Washington.

III. *Committee for the organization of eclipse observations:*

A brief report for the committee was presented by Comte de la Baume Pluvinel, referring to the two eclipses which had occurred since the committee was appointed. Additional remarks on the eclipse expedition to Flint Island were made by W. W. Campbell of the Lick Observatory, who also exhibited photographs obtained by expeditions from the Lick Observatory at this and other eclipses.

IV. *Committee on the determination of solar rotation by means of the displacement of lines:*

The report was presented by Walter S. Adams of the Mount Wilson Solar Observatory.

The committee recommended that a larger number of lines be employed, both of the same elements and of different elements. Different parts of the spectrum were assigned to the various observers, and it was recommended that one portion of the spectrum be observed in common by all taking part in the work.

V. *Committee on work with the spectroheliograph:*

The report was presented by Edwin B. Frost of the Yerkes Observatory. The resolutions suggested by the committee were adopted.

Supplementary reports of work accomplished were presented from the observatories of Meudon, Catania, Tortosa, the Yerkes Observatory, and the Solar Physics Observatory in London, which included the work of the Kodaikánal Observatory.

After a general discussion at the final session it was

VOTED that the Union extend its activity so as to include general astrophysics.

VOTED that a committee be elected to consider and report on the question of the classification of stellar spectra.

This committee was subsequently elected by the members present, and was organized with Edward C. Pickering as chairman, and Frank Schlesinger of the Allegheny Observatory as secretary.

VOTED that the next meeting of the Union be held at Bonn in the summer of 1913, the exact date to be determined later.

The various committees of the Union were elected, with some additions and changes, and the two retiring members of the Executive Committee were re-elected.

The following list contains the names of most of the members of the Union and guests present at this meeting. F.

MEMBERS AND INVITED GUESTS PRESENT AT THE FOURTH CONFERENCE

J. S. Ames, Johns Hopkins University, Baltimore, Md.
Charles G. Abbot, Smithsonian Astrophysical Observatory, Washington, D.C.
Walter S. Adams, Mt. Wilson Solar Observatory
Harold D. Babcock, Mt. Wilson Solar Observatory
O. Backlund, Observatoire de Poulkovo, Poulkovo, Russia
E. E. Barnard, Yerkes Observatory, Williams Bay, Wis.
A. Belopolsky, Observatoire de Poulkovo, Poulkovo, Russia
Jean Bosler, Observatoire de Meudon, Meudon, France
F. P. Brackett, Pomona College, Claremont, Cal.
Miss Cora G. Burwell, Mt. Wilson Solar Observatory
W. W. Campbell, Lick Observatory, Mount Hamilton, Cal.
C. A. Chant, University of Toronto, Toronto, Canada
Henri Chrétien, Observatoire de Nice, Nice, France
P. R. Cirera, Observatorio del Ebro, Tortosa, Spain
W. W. Coblentz, Bureau of Standards, Washington, D.C.
A. L. Cortie, Stonyhurst College Observatory, Lancashire, England
A. Cotton, Ecole Normale Supérieure, Paris, France
H. Deslandres, Observatoire de Meudon, Meudon, France
N. Donitch, Observatoire de l'Université, St. Petersburg, Russia
Frank L. Drew, Mt. Wilson Solar Observatory
F. W. Dyson, Royal Observatory, Edinburgh, Scotland
Ferdinand Ellerman, Mt. Wilson Solar Observatory
P. Eversheim, University of Bonn, Bonn, Germany
Charles Fabry, Université de Marseille, Marseille, France
Edward A. Fath, Mt. Wilson Solar Observatory
Mrs. W. P. Fleming, Harvard College Observatory, Cambridge, Mass.
F. E. Fowle, Smithsonian Astrophysical Observatory, Washington, D.C.

A. Fowler, Imperial College of Science and Technology, South Kensington, London, England
Philip Fox, Dearborn Observatory, Evanston, Ill.
Edwin B. Frost, Yerkes Observatory, Williams Bay, Wis.
Henry G. Gale, University of Chicago, Chicago, Ill.
L. H. Gilmore, Throop Polytechnic Institute, Pasadena, Cal.
Miss Clementina D. Griffin, Mt. Wilson Solar Observatory
George E. Hale, Mt. Wilson Solar Observatory
M. Hamy, Observatoire de Paris, Paris, France
J. Hartmann, Königliche Sternwarte, Göttingen, Germany
K. Haussmann, Technische Hochschule, Aachen, Germany
J. v. Hepperger, Imperial Observatory, Vienna, Austria
E. H. Hills, 32 Prince's Garden, London, England
W. J. Humphreys, U.S. Weather Bureau, Washington, D.C.
P. Idrac, Observatoire de Meudon, Meudon, France
J. C. Kapteyn, Astronomical Laboratory, Groningen, Holland
H. Kayser, University of Bonn, Bonn, Germany
Arthur S. King, Mt. Wilson Solar Observatory
H. Konen, Physikalisches Institut, Münster, Germany
F. Küstner, Königliche Sternwarte, Bonn, Germany
C. O. Lampland, Lowell Observatory, Flagstaff, Arizona
Sir Joseph Larmor, Royal Society, London, England
Miss Jennie B. Lasby, Mt. Wilson Solar Observatory
A. O. Leuschner, University of California, Berkeley, Cal.
H. C. Lord, McMillin Observatory, Columbus, Ohio
A. G. McAdie, U.S. Weather Bureau, San Francisco, Cal.
J. D. Maddrill, International Latitude Observatory, Ukiah, Cal.
Walter M. Mitchell, Detroit Observatory, Ann Arbor, Mich.
H. F. Newall, University Observatory, Cambridge, England
F. G. Pease, Mt. Wilson Solar Observatory
H. B. Perkins, Throop Polytechnic Institute, Pasadena, Cal.
E. C. Pickering, Harvard College Observatory, Cambridge, Mass.
J. S. Plaskett, Dominion Observatory, Ottawa, Canada
Comte A. de la Baume Pluvinel, 7 Rue de la Baume, Paris, France
E. Pringsheim, University of Breslau, Breslau, Germany
P. Puiseux, Observatoire de Paris, Paris, France
A. Riccò, Osservatorio astrofisico, Catania, Sicily
G. W. Ritchey, Mt. Wilson Solar Observatory
A. L. Rotch, Blue Hill Observatory, Hyde Park, Mass.
Henry Norris Russell, Princeton University, Princeton, N.J.
J. R. Rydberg, University of Lund, Lund, Sweden
Charles E. St. John, Mt. Wilson Solar Observatory
Fernando Sanford, Leland Stanford Jr. University, Palo Alto, Cal.
Frank Schlesinger, Allegheny Observatory, Allegheny, Pa.

Arthur Schuster, Victoria Park, Manchester, England
K. Schwarzschild, Astrophysikalisches Observatorium, Potsdam, Germany
F. H. Seares, Mt. Wilson Solar Observatory
V. M. Slipher, Lowell Observatory, Flagstaff, Arizona
Frederick Slocum, Yerkes Observatory, Williams Bay, Wis.
Miss Ruth E. Smith, Mt. Wilson Solar Observatory
S. W. Stratton, Bureau of Standards, Washington, D.C.
H. Struve, Königliche Sternwarte, Berlin, Germany
S. D. Townley, Leland Stanford Jr. University, Palo Alto, Cal.
H. H. Turner, University Observatory, Oxford, England
Miss Louise Ware, Mt. Wilson Solar Observatory
Miss Phoebe Waterman, Mt. Wilson Solar Observatory
F. R. Watson, University of Illinois, Champaign, Ill.
H. C. Wilson, Goodsell Observatory, Northfield, Minn.
A. Wolfer, Sternwarte des eidgenossischen Polytechnikums, Zurich, Switzerland

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Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

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